AP Calculus BC – Unit 4, Part 2 Required Practice **Name:** answers

4.5 day 1 – Required Practice

Evaluate the integral.

#1. $\int x^2\sqrt{2x^3-3}\,dx$

$\frac{1}{9}(2x^3-3)^{3/2}+C$

#2. $\int x^3\left(1-x^4\right)^5\,dx$

$-\frac{1}{24}(1-x^4)^6+C$

#3. $\int \frac{x^2}{\sqrt{1-x}}\,dx$

$-2(1-x)^{1/2}+\frac{4}{3}(1-x)^{3/2}-\frac{2}{5}(1-x)^{5/2}+C$

#4. $\int \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

$-\frac{2}{3}\left(1+\frac{1}{x}\right)^{3/2}+C$

#5. $\int \frac{\sin x}{1+\cos^2 x}\,dx$

$-\arctan(\cos x)+C$

#6. $\int \sec x \tan x \sqrt{1+\sec x}\,dx$

$\frac{2}{3}(1+\sec x)^{3/2}+C$

Showing the u-substitution

#7. $\int 3(3x-2)^5\,dx$

$\frac{(3x-2)^6}{6}+C$

Reverse Chain Rule

#8. $\int 3(3x-2)^5\,dx$

$\frac{(3x-2)^6}{6}+C$

#9. $\int \frac{x^2}{\sqrt{4-x^3}}\,dx$

$-\frac{2}{3}(4-x^3)^{1/2}+C$

#10. $\int \frac{x^2}{\sqrt{4-x^3}}\,dx$

$-\frac{2}{3}(4-x^3)^{1/2}+C$

Showing the u-substitution

#11. $\int (2x^2 - 3x)^4 (4x-3)\, dx$

$\frac{(2x^2-3x)^5}{5} + C$

#13. $\int \cos(x^2+3x)(6x+9)\, dx$

$3\sin(x^2+3x)+C$

#15. $\int \cos(5x)\, dx$

$\frac{1}{5}\sin(5x)+C$

Reverse Chain Rule

#12. $\int (2x^2 - 3x)^4 (4x-3)\, dx$

$\frac{(2x^2-3x)^5}{5} + C$

#14. $\int \cos(x^2+3x)(6x+9)\, dx$

$3\sin(x^2+3x)+C$

#16. $\int \cos(5x)\, dx$

$\frac{1}{5}\sin(5x)+C$

Evaluate using u-substitution or Reverse Chain Rule (whichever you prefer)

#17. $\int 6(1+6x)^4\,dx$

$\frac{(1+6x)^5}{5}+c$

#18. $\int \frac{x}{\sqrt{1-x^2}}\,dx$

$-\sqrt{1-x^2}+c$

#19. $\int e^{-x}\sec^2\left(e^{-x}\right)dx$

$-\tan(e^{-x})+C$

#20. $\int \ln\left(e^{2x-1}\right)dx$

x^2-x+C

4.5 day 2 – Required Practice

Evaluate the definite integral.

#1. $\int_1^3 x\sqrt{3x^2-2}\,dx$

$\frac{1}{9}\left[(25)^{3/2}-(1)^{3/2}\right]$ or $\frac{1}{9}\left[(3(3)^2-2)^{3/2}-(3(1)^2-2)^{3/2}\right]$

#2. $\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

$\frac{1}{2}\left[(\sin^{-1}(\frac{1}{2}))^2-(\sin^{-1}(0))^2\right]$

Evaluate the definite integral.

#3. $\int_1^2 2x^2\sqrt{x^3+1}\,dx$

$\frac{4}{9}\left[(9)^{3/2}-(2)^{3/2}\right]$

#4. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx$

$\frac{1}{2}\left[\sqrt{9}-\sqrt{1}\right]$

$(=1)$

#5. $\int_1^3 \frac{e^{\left(\frac{3}{x}\right)}}{x^2}\,dx$

$-\frac{1}{3}\left[e^1-e^3\right]$

#6. $\int_1^9 \frac{1}{\sqrt{x}\left(1+\sqrt{x}\right)^2}\,dx$

$2\left[-\frac{1}{4}-\left(-\frac{1}{2}\right)\right]$

4.6 – Required Practice

#1. $\int \frac{1}{4+x^2}\,dx$

$\frac{1}{2}\arctan\left(\frac{x}{2}\right)+C$

#2. $\int \frac{x+2}{\sqrt{4-x^2}}\,dx$

$-(4-x^2)^{1/2}+2\arcsin\left(\frac{x}{2}\right)+C$

#3. $\int \frac{1}{x^2-4x+7}\,dx$

$\frac{1}{\sqrt{3}}\arctan\left(\frac{x-2}{\sqrt{3}}\right)+C$

#4. $\int \tan(2x)\,dx$

$-\frac{1}{2}\ln|\cos(2x)|+C$

#5. $\int x^3 \tan(x^4)\,dx$

$-\frac{1}{4}\ln|\cos(x^4)|+C$

#6. $\int \frac{1}{1+e^x}\,dx$

$x-\ln|1+e^x|+C$

Evaluate the integral.

#7. $\int \frac{x^4 - 3x^2}{\sqrt{x}} \, dx$

$\frac{2}{9} x^{9/2} - 3\left(\frac{2}{5}\right) x^{5/2} + C$

#8. $\int \frac{x-5}{\sqrt{16-9x^2}} \, dx$

$-\frac{1}{18}(2)(16-9x^2)^{1/2} - \frac{5}{3} \arcsin\left(\frac{3x}{4}\right) + C$

#9. $\int x^2 \tan\left(x^3 - 2\right) dx$

$-\frac{1}{3} \ln \left|\cos(x^3-2)\right| + C$

#10. $\int \left(\tan^2(3x) + 1\right) dx$

$\frac{1}{3} \tan(3x) + C$

#11. $\int \frac{1}{x^2+6x+12}\,dx$

$\frac{1}{\sqrt{3}} \arctan\left(\frac{x-3}{\sqrt{3}}\right)+c$

#12. $\int \frac{1}{x^2-2x+5}\,dx$

$\frac{1}{2}\arctan\left(\frac{x-1}{2}\right)+c$

#13. $\int \frac{x^2-4x+2}{x^2+2}\,dx$

$x-2\ln(x^2+2)+c$

#14. $\int \frac{2}{e^{-x}+1}\,dx$

$2x+2\ln|e^{-x}+1| +c$

4.7 day 1 – Required Practice

Evaluate the integral.

#1. $\int e^{(3x)}\,dx$

$\frac{1}{3}e^{(3x)}+C$

#2. $\int 3xe^{x}\,dx$

$3xe^{x}-3e^{x}+C$

#3. $\int \ln(x)\,dx$

$x\ln x-x+C$

#4. $\int x\sin(4x)\,dx$

$-\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)+C$

#5. $\int 2x^3 \cos\left(x^2\right) dx$

$x^2 \sin(x^2) + \cos(x^2) + C$

#6. $\int x^2 \sin(x)\, dx$

$-x^2\cos x + 2x\sin x + 2\cos x + C$

Evaluate the integral.

#7. $\int xe^x \, dx$

$xe^x - e^x + C$

#8. $\int x^3 \ln(x) \, dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

#9. $\int \ln(5x) \, dx$

$x \ln(5x) - x + C$

#10. $\int x^2 \cos(3x) \, dx$

$\frac{1}{3}x^2 \sin(3x) + \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + C$

#11. $\int t\ln(t+1)\,dt$

$\frac{1}{2}t^2\ln(t+1)-\frac{1}{2}\left[\frac{1}{2}(t+1)^2-2(t+1)+\ln|t+1|\right]+C$

4.7 day 2 – Required Practice

Evaluate the integral.

#1. $\int e^{4x}\cos(2x)\,dx$

$\frac{4}{5}\left[\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)\right]+C$

The usual method...

#2. $\int x^2 e^{2x}\,dx$

$\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$

The tabular method...

#3. $\int x^2 e^{2x}\,dx$

$\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$

#4. $\int x^6 e^{x}\,dx$

$x^6e^x-6x^5e^x+30x^4e^x-120x^3e^x+360x^2e^x-720xe^x+720e^x+C$

Evaluate the integral.

#5. $\int e^{5x} \cos(x)\, dx$

$\frac{1}{26}\left(e^{5x} \sin x + 5e^{5x} \cos x\right) + C$

#6. $\int x^2 e^{2x}\,dx$

$\frac{1}{2}x^2e^{2x}-\frac{1}{2}xe^{2x}+\frac{1}{4}e^{2x}+C$

#7. $\int x^3 \sin(x)\,dx$

$-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+C$

#8. $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

a) by integration by parts using $u=x^2$

$x^2\sqrt{4+x^2}-\frac{2}{3}(4+x^2)^{3/2}+C$

b) by u-substitution using $u=4+x^2$

$\frac{1}{3}(4+x^2)^{3/2}-4(4+x^2)^{1/2}+C$

4.8 – Required Practice

#1. $\int \sin^3 x \cos^3 x \, dx$

$\frac{1}{4} \sin^4 x - \frac{1}{6} \sin^6 x + C$

#2. $\int \tan^2 x \sec^2 x \, dx$

$\frac{1}{3} \tan^3 x + C$

#3. $\int \cos x \, dx$

$\sin x + C$

#4. $\int \tan x \, dx$

$-\ln|\cos x| + C$

#5. $\int \sec x \, dx$

$\ln|\sec x + \tan x| + C$

#6. $\int \cos^2 x \, dx$

$\frac{1}{2}x + \frac{1}{4}\sin(2x) + C$

#7. $\int \sin^2 x \, dx$

$\frac{1}{2}x - \frac{1}{4}\sin(2x) + C$

#8. $\int \tan^4 x \, dx$

$\frac{1}{3}\tan^3 x - \tan x + x + C$

#9. $\int \cot^5 x \sin^4 x \, dx$

$\ln|\sin x| - \sin^2 x + \frac{1}{4}\sin^4 x + C$

Evaluate:

#10. $\int \sin^3 x \cos^4 x \, dx$

$-\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$

#11. $\int \sin^7(2x)\cos(2x)\, dx$

$-\frac{1}{4}\cos^2(2x) + \frac{3}{8}\cos^4(2x) - \frac{1}{4}\cos^6(2x) + \frac{1}{16}\cos^8(2x) + C$

#12. $\int \cos^2(3x)\, dx$

$\frac{1}{2}x + \frac{1}{12}\sin(6x) + C$

#13. $\int 6\sec(4x)\, dx$

$\frac{3}{2}\ln\left|\sec(4x) + \tan(4x)\right| + C$

#14. $\int \tan^3(2x)\sec^3(2x)\,dx$

$\frac{1}{10}\sec^5(2x) - \frac{1}{6}\sec^3(2x) + C$

#15. $\int \frac{\tan^2(x)}{\sec(x)}\,dx$

$\ln|\sec x + \tan x| + \sin x + C$

#16. $\int \tan^3(2x)\,dx$

$\frac{1}{4}\sec^2(2x) + \ln|\cos(2x)| + C$

4.9 – Required Practice

#1. $\int \frac{\sqrt{x^2-9}}{x}\,dx$

$$3\left(\frac{\sqrt{x^2-9}}{3}\right) - 3\arccos\left(\frac{3}{x}\right) + C$$

#2. $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$

$$-64\left(\frac{\sqrt{16-x^2}}{4}\right) + \frac{64}{3}\left(\frac{\sqrt{16-x^2}}{4}\right)^3 + C$$

#3. $\int x^3\sqrt{x^2+4}\,dx$

$$\frac{32}{5}\left(\frac{\sqrt{x^2+4}}{2}\right)^5 - \frac{32}{3}\left(\frac{\sqrt{x^2+4}}{2}\right)^3 + c$$

#4. $\int \frac{\sqrt{x^6-4}}{x}\,dx$

$$\frac{2}{3}\left(\frac{\sqrt{x^6-4}}{2}\right) - \frac{2}{3}\arccos\left(\frac{2}{x^3}\right) + c$$

Evaluate the integral.

#5. $\int \frac{\sqrt{16-x^2}}{x}\,dx$

$-4\ln\left|\frac{4}{x}+\frac{\sqrt{16-x^2}}{x}\right|+4\sqrt{16-x^2}+C$

#6. $\int \frac{\sqrt{x^2-25}}{x}\,dx$

$5\left(\frac{\sqrt{x^2-25}}{5}\right)-5\arccos\left(\frac{5}{x}\right)+C$

#7. $\int \frac{9x^3}{\sqrt{1+x^2}}\,dx$

$3\left(\sqrt{1+x^2}\right)^3 - 9\sqrt{1+x^2} + C$

#8. $\int \sqrt{16-4x^2}\,dx$

$-4\arccos\left(\frac{1}{2}x\right) - 4\left(\frac{\sqrt{16-4x^2}}{4}\right)\left(\frac{1}{2}x\right) + C$

4.10 – Required Practice

#1. $\int \frac{3x-5}{x^2+6x-7}dx$

$\frac{13}{4}\ln|x+7| - \frac{1}{4}\ln|x-1| + C$

#2. $\int \frac{1}{x^2-5x+6}dx$

$-\ln|x-2| + \ln|x-3| + C$

$= \ln\left|\frac{x-3}{x-2}\right| + C$

#3. $\int \frac{x^3 - x + 3}{x^2 + x - 2}\, dx$

$\frac{1}{2}x^2 - x + \ln|x+2| + \ln|x-1| + C$

#4. $\int \frac{1}{(x-5)^3 (x^2+x+1)^2}\, dx$ $= \frac{A}{x-5} + \frac{B}{(x-5)^2} + \frac{C}{(x-5)^3} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{(x^2+x+1)^2}$

#5. $\int \frac{5x+3}{x^2+x-2}\,dx$

$\frac{7}{3}\ln|x+2| + \frac{8}{3}\ln|x-1| + C$

#6. $\int \frac{3x-5}{x^2+2x-8}\,dx$

$\frac{1}{6}\ln|x+4| + \frac{17}{6}\ln|x-2| + C$

#7. $\int \frac{6x-7}{x^2-3x-4}\,dx$

$\frac{13}{5}\ln|x+1| + \frac{17}{5}\ln|x-4| + C$

Evaluate the integral.

#8. $\int \frac{1}{x^2-9}dx$

$\frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3| + c$

#9. $\int \frac{5+x}{2x^2-7x-4}dx$

$\ln|x-4| - \frac{1}{2}\ln|2x+1| + c$

#10. $\int \frac{x^2+12x+12}{x^3-4x}\,dx$

$-\ln|x| + 6\ln|x-2| - 4\ln|x+2| + C$

4.11 – Required Practice

#1. $\int_{1}^{\infty} \frac{1}{x^2}\,dx$

1

#2. $\int_{1}^{\infty} \frac{1}{x}\,dx$

diverges

#3. $\int_{0}^{1} \frac{1}{\sqrt[3]{x}}\,dx$

$\frac{3}{2}$

#4. $\int_0^3 \frac{2x-1}{x^2-x-2}\,dx = \int_0^3 \frac{2x-1}{(x+1)(x+2)}\,dx$

typo $(x-2)$

diverges

#5. $\int_0^\infty \cos(x)\,dx$

diverges

#6. $\int_{-\infty}^{1} xe^{2x}\,dx$

$\frac{1}{4}e^{2}$

Evaluate the integral or state that it diverges.

#7. $\int_0^4 \frac{1}{\sqrt{x}}\,dx$

4

#8. $\int_0^2 \frac{1}{(x-1)^2}\,dx$

diverges

#9. $\int_1^\infty \frac{1}{x^3}\,dx$

$\frac{1}{2}$

#10. $\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$

diverges

Unit 4 Part 2 Test Review

Evaluate the integral or state that it diverges.

#1. $\int \sin\theta \, d\theta$

#2. $\int 7x^3\left(3x^4+6\right)^7 \, dx$

#3. $\int \frac{1+\cos x}{\sin x} \, dx$

#4. $\int x\sin(2x)\,dx$

#5. $\int_1^2 e^{3x} \, dx$

#6. $\int_3^\infty \frac{1}{x(\ln x)^2} \, dx$

#7. $\int \sin^3 x \cos x \, dx$

#8. $\int \frac{1}{(x-4)(x+5)} \, dx$

#9. $\int \frac{1}{\sqrt{x^2+16}} \, dx$

#10. $\int e^{\left(x+e^x\right)} \, dx$

#11. $\int 5\csc^2 x \, dx$

#12. $\int e^x \, dx$

#13. $\int x^2 \sin(x) \, dx$

#14. $\int \frac{3}{\sqrt{9-(x+5)^2}} \, dx$

#15. $\int_1^3 \frac{2}{x-2} \, dx$

#16. $\int 2x^3\,dx$

#17. $\int \frac{2}{(x-10)^2+36}\,dx$

#18. $\int \sec x \tan x\,dx$

#19. $\int x^2 e^x\,dx$

#20. $\int (4x+6)e^{(x^2+3x)}\,dx$

#21. $\int_0^5 \frac{1}{(x-1)^{1/5}}\,dx$

#22. $\int \frac{3x^2-2}{x^3-2x-8}\,dx$

#23. $\int \sin^3 x \cos^5 x\,dx$

#24. $\int 2x^{-1}\,dx$

#25. $\int \sec x\,dx$

#26. $\int \csc x\,dx$

#27. $\int_4^\infty \frac{x}{x^{7/2}}\,dx$

#28. $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$

#29. $\int \sin^2 x\,dx$

#30. $\int \cos^2 x\,dx$

#31. $\int \frac{1}{x^2-5x-14}\,dx$

#32. $\int \frac{1}{x^2-12x+38}\,dx$

#33. $\int \frac{1-\sin^2 x}{\cos x}\, dx$

#34. $\int xe^x\, dx$

#35. $\int \csc^4 x \cot^3 x\, dx$

#36. $\int_3^{\infty} x \ln\left(x^2\right) dx$

#37. $\int \frac{\sec x}{\tan^2 x + 1}\, dx$

#38. $\int 7e^{5x}\, dx$

#39. $\int \cot x\, dx$

#40. $\int 2x^3 \cos\left(x^2\right) dx$

#41. $\int_0^{\infty} \frac{x^2}{\left(1-x^3\right)^2}\, dx$

#42. $\int 3\sec^2 x\, dx$

#43. $\int \frac{2t}{(t-3)^2}\, dt$

#44. $\int \tan x\, dx$

#45. $\int \csc x \cot x\, dx$

#46. $\int x \ln(x)\, dx$

#47. $\int \frac{\sin x + \sec x}{\tan x}\, dx$

#48. $\int 3x \ln\left(x^2\right) dx$

#49. $\int \cos x\left(1+\sin^2 x\right) dx$

#50. $\int \left(x^3 + e^{4x} + \cos(x)\right) dx$

Unit 4 Part 2 test review — SOLUTIONS

#1 $\int \sin\theta \, d\theta = \boxed{-\cos\theta + C}$

#2 $\int 7x^3(3x^4+6)^7 dx$

u-sub
$u = 3x^4+6$
$\frac{du}{dx} = 12x^3$
$du = 12x^3 dx$
$x^3 dx = \frac{1}{12} du$

$7\int u^7 \left(\frac{1}{12} du\right)$

$\frac{7}{12}\int u^7 du$

$\frac{7}{12}\left(\frac{1}{8}u^8\right) + c$

$\boxed{\frac{7}{12}\left(\frac{1}{8}\right)(3x^4+6)^8 + C}$

#3 $\int \frac{1+\cos x}{\sin x} dx$ (common denominator)

$\int \frac{1}{\sin x} dx + \int \frac{\cos x}{\sin x} dx$

$\int \csc x \, dx + \int \frac{\cos x}{\sin x} dx$

u-sub
$u = \sin x$
$\frac{du}{dx} = \cos x$
$du = \cos x \, dx$

$\int \csc x \, dx + \int \frac{1}{u} du$

$\boxed{\ln|\csc x - \cot x| - \ln|\sin x| + c}$

(memorize $\int \sec x \, dx = \ln|\sec x + \tan x| + C$
and $\int \csc x \, dx = \ln|\csc x - \cot x| + c$)

#4) $\int x \sin(2x)\,dx$

by parts

$u = x \qquad dv = \sin(2x)\,dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int \sin(2x)\,dx$

$du = dx \qquad v = -\frac{1}{2}\cos(2x)$

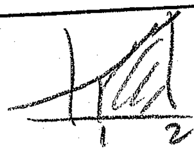
$uv - \int v\,du$

$-\frac{1}{2}x\cos(2x) - \int \left(-\frac{1}{2}\cos(2x)\right)dx$

$-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx$

$\boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

#5) $\int_1^2 e^{3x}\,dx$



no issues w/ converges
(not an improper integral)

$\left[\frac{e^{3x}}{3}\right]_1^2$

$\boxed{\frac{1}{3}e^{3(2)} - \frac{1}{3}e^{3(1)}}$

(don't simplify on tests unless the problem say to ☺)

#6 $\int_3^{\infty} \frac{1}{x(\ln x)^2} dx$

$= \lim_{b \to \infty} \int_3^b \frac{1}{x(\ln x)^2} dx$

$\int \frac{1}{x(\ln x)^2} dx$

u-sub

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$\int u^{-2} du$

$du = \frac{1}{x} dx$

$\frac{u^{-1}}{-1} + C = -\frac{1}{u} + C = -\frac{1}{\ln x} + C$

$\lim_{b \to \infty} \left[-\frac{1}{\ln(x)} \right]_3^b$

$\lim_{b \to \infty} \left[-\frac{1}{\ln(b)} \right] - \left[-\frac{1}{\ln(3)} \right]$

$0 + \frac{1}{\ln(3)} = \boxed{\frac{1}{\ln(3)}}$

#7 $\int \sin^3 x \cos x \, dx$

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$\int u^3 du$

$\frac{1}{4} u^4 + C$

$\boxed{\frac{1}{4} \sin^4 x + C}$

#8 $\int \frac{1}{(x-4)(x+5)}\,dx$

partial fractions

$$\frac{1}{(x-4)(x+5)} = \frac{A}{x-4} + \frac{B}{x+5}$$

$$A(x+5)+B(x-4)=1$$

$$Ax+5A+Bx-4B=1$$

$$(A+B)x+(5A-4B)=(0)x+(1)$$

$$\begin{cases} A+B=0 \\ 5A-4B=1 \end{cases}$$

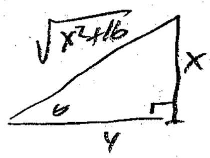
$$\begin{array}{r} 4A+4B=0 \\ 5A-4B=1 \\ \hline 9A=1 \end{array}, \quad A=\frac{1}{9}, \quad A+B=0, \quad B=-\frac{1}{9}$$

$$\frac{1}{9}\int \frac{1}{x-4}\,dx - \frac{1}{9}\int \frac{1}{x+5}\,dx$$

$$\boxed{\frac{1}{9}\ln|x-4| - \frac{1}{9}\ln|x+5| + C}$$

#9 $\int \frac{1}{\sqrt{x^2+16}}\,dx$

trig sub



$\tan\theta = \frac{x}{4}$ $\qquad$ $\cos\theta = \frac{4}{\sqrt{x^2+16}}$

$x = 4\tan\theta$ $\qquad$ $\sqrt{x^2+16} = \frac{4}{\cos\theta} = 4\sec\theta$

$\frac{dx}{d\theta} = 4\sec^2\theta$ $\qquad$ $\sec\theta = \frac{\sqrt{x^2+16}}{4}$

$dx = 4\sec^2\theta\,d\theta$

$$\int \frac{1}{4\sec\theta}\,4\sec^2\theta\,d\theta$$

$$\int \sec\theta\,d\theta$$

$$\ln|\sec\theta+\tan\theta|+C$$

$$\boxed{\ln\left|\frac{\sqrt{x^2+16}}{4}+\frac{x}{4}\right|+C}$$

(← you must return to the original variable)

#10 $\int e^{(x+e^x)}dx$

u-sub

$u = x+e^x$

$\frac{du}{dx} = \underbrace{1+e^x}_{\text{not in integrand}}$

but $\int e^{(x+e^x)}dx = \int e^x e^{(e^x)} dx$

suggests we could use

$u = e^x$

$\frac{du}{dx} = e^x$

$du = e^x dx$

$= \int e^u du$

$= e^u$

$= \boxed{e^{(e^x)} + c}$

#11 $\int 5\csc^2 x\, dx$

$\boxed{-5\cot x + C}$

#12 $\int e^x dx$

$\boxed{e^x + c}$

#13 $\int x^2 \sin x\,dx$ by parts (twice)

$u = x^2 \quad dv = \sin x\,dx$

$\frac{du}{dx} = 2x \quad \int dv = \int \sin x\,dx$

$du = 2x\,dx \quad v = -\cos x$

$uv - \int v\,du$

$-x^2\cos x - \int(-\cos x)2x\,dx$

$-x^2\cos x + 2\left[\int x\cos x\,dx\right]$

$u = x \quad dv = \cos x\,dx$

$\frac{du}{dx} = 1 \quad \int dv = \int \cos x\,dx$

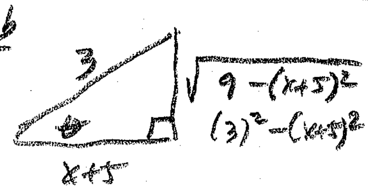
$du = dx \quad v = \sin x$

$-x^2\cos x + 2\left[uv - \int v\,du\right]$

$-x^2\cos x + 2\left[x\sin x - \int \sin x\,dx\right]$

$\boxed{-x^2\cos x + 2\left[x\sin x + \cos x\right] + C}$

#14 $\int \frac{3}{\sqrt{9-(x+5)^2}}\,dx$ trig sub



$\int \frac{3\,(-3)\sin\theta}{3\sin\theta}\,d\theta$

$\cos\theta = \frac{x+5}{3} \qquad \sin\theta = \frac{\sqrt{9-(x+5)^2}}{3}$

$x+5 = 3\cos\theta \qquad \sqrt{9-(x+5)^2} = 3\sin\theta$

$\int -3\,d\theta$

$x = 3\cos\theta - 5$

$\frac{dx}{d\theta} = -3\sin\theta$

$-3\theta + C$

$dx = -3\sin\theta\,d\theta$

$\boxed{-3\arccos\left(\frac{x+5}{3}\right) + C}$

$\theta = \arccos\left(\frac{x+5}{3}\right)$

—or—

memorized form! $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right) + C$

$3\int \frac{1}{\sqrt{a^2-u^2}}\,du \qquad u = x+5 \quad a = 3$

$du = dx$

$\boxed{3\arcsin\left(\frac{x+5}{3}\right) + C}$

either is correct :)

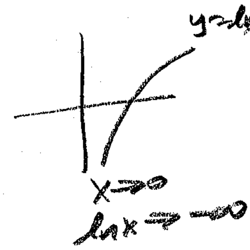
#15 $\int_1^3 \frac{2}{x-2}dx$ vertical asymptote at $x=2$ is in the integral (improper)

$$= \lim_{b\to 2^-} \int_1^b \frac{2}{x-2}dx + \lim_{d\to 2^+} \int_d^3 \frac{2}{x-2}dx \qquad \left(\int \frac{2}{x-2}dx \quad 2\ln|x-2|+C \right)$$

$$= \lim_{b\to 2^-} \Big[2\ln|x-2|\Big]_1^b + \lim_{d\to 2^+} \Big[2\ln|x-2|\Big]_d^2$$

$$= \lim_{b\to 2^-} 2\ln|b-2| - 2\ln|x-1| + 2\ln|2-2| - \lim_{d\to 2^+} [2\ln|d-2|]$$

$$-\infty \qquad \boxed{\text{diverges}}$$



#16 $\int 2x^3 dx$

$$\boxed{2\frac{x^4}{4} + C}$$

#17 $\int \frac{2}{(x-10)^2+36} dx$ $\quad u = x-10 \quad a=6$

$\frac{du}{dx} = 1$

$du = dx$

$$2\int \frac{1}{u^2+a^2} du$$

$$\left(\int \frac{1}{a^2+u^2} du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C \right)$$

$$\boxed{2\left(\tfrac{1}{6}\right)\arctan\left(\frac{x-10}{6}\right)+C}$$

#18 $\int \sec x \tan x \, dx = \boxed{\sec x + C}$

#19 $\int x^2 e^x \, dx$ <u>by parts</u> (twice)

$u = x^2 \quad dv = e^x dx$

$\frac{du}{dx} = 2x \quad \int dv = \int e^x dx$

$du = 2x dx \quad v = e^x$

$uv - \int v du$

$x^2 e^x - 2\left[\int x e^x dx\right]$

$u = x \quad dv = e^x dx$

$\frac{du}{dx} = 1 \quad \int dv = \int e^x dx$

$du = dx \quad v = e^x$

$x^2 e^x - 2\left[uv - \int v du\right]$

$x^2 e^x - 2\left(x e^x - \int e^x dx\right)$

$\boxed{x^2 e^x - 2\left[x e^x - e^x\right] + C}$

#20 $\int (4x+6) e^{(x^2+3x)} dx$ <u>u-sub</u>

$u = x^2 + 3x$

$\frac{du}{dx} = 2x + 3$

$du = (2x+3) dx$

$\int 2(2x+3) e^{(x^2+3x)} dx$

$2 \int e^u du$

$2e^u + C$

$\boxed{2e^{(x^2+3x)} + C}$

#21 $\int_0^5 \frac{1}{(x-1)^{1/5}}\,dx$ vertical asymptote at $x=1$ is w/in the interval (improper)

$= \lim_{b \to 1^-} \int_0^b (x-1)^{-1/5}dx + \lim_{d \to 1^+} \int_d^5 (x-1)^{-1/5}dx$

$\int (x-1)^{-1/5}dx$ $u=x-1$

$du=dx$

$\int u^{-1/5}du$

$\frac{u^{4/5}}{(4/5)} = \frac{5}{4}u^{4/5}+c$

$\frac{5}{4}(x-1)^{4/5}+c$

$= \lim_{b \to 1^-}\left[\frac{5}{4}(x-1)^{4/5}\right]_0^b + \lim_{d \to 1^+}\left[\frac{5}{4}(x-1)^{4/5}\right]_d^5$

$= \lim_{b \to 1^-}\left(\frac{5}{4}(b-1)^{4/5}\right) - \frac{5}{4}(0-1)^{4/5} + \frac{5}{4}(5-1)^{4/5} - \lim_{d \to 1^+}\left(\frac{5}{4}(d-1)^{4/5}\right)$

$0 - \frac{5}{4} + \frac{5}{4}(4)^{4/5} - 0$

$= \boxed{-\frac{5}{4} + \frac{5}{4}(4)^{4/5}}$

#22 $\int \frac{3x^2-2}{x^3-2x-8}\,dx$

u-sub

$u=x^3-2x-8$

$\frac{du}{dx}=3x^2-2$

$du=(3x^2-2)dx$

$\int \frac{1}{u}\,du$

$\ln|u|+c$

$\boxed{\ln|x^3-2x-8|+c}$

#23 $\int \sin^3 x \cos^5 x \, dx$

$\int \sin^2 x \cos^5 x \, \underline{\sin x \, dx}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

$\sin x \, dx = -du$

$\int (1-\cos^2 x) \cos^5 x \sin x \, dx$

$\int (1-u^2) u^5 (-du)$

$-\int (u^5 - u^7) \, du$

$-\frac{1}{6}u^6 + \frac{1}{8}u^8 + C$

$\boxed{-\frac{1}{6}\cos^6 x + \frac{1}{8}\cos^8 x + C}$

#24 $\int 2x^{-1} dx = 2\int \frac{1}{x} dx = \boxed{2\ln|x| + C}$

#25 $\int \sec x \, dx$

$= \boxed{\ln|\sec x + \tan x| + C}$

memorize those

#26 $\int \csc x \, dx$

$= \boxed{\ln|\csc x - \cot x| + C}$

#27 $\displaystyle\int_4^{\infty} \frac{x}{x^{7/2}}dx$

$= \displaystyle\lim_{b\to\infty} \int_4^{b} x^{-5/2}\,dx$

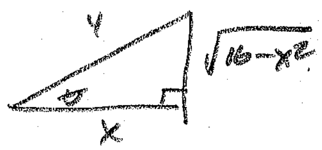
$= \displaystyle\lim_{b\to\infty} \left[\left(-\frac{2}{3}\right) x^{-3/2}\right]_4^b$

$= \displaystyle\lim_{b\to\infty} \left[\frac{-2}{3x^{3/2}}\right]_4^b$

$= \displaystyle\lim_{b\to\infty} \left[\frac{-2}{3(b)^{3/2}}\right] - \left(\frac{-2}{3(4)^{3/2}}\right)$

$0 + \dfrac{2}{3(4)^{3/2}} = \boxed{\dfrac{2}{3(4)^{3/2}}}$

#28 $\displaystyle\int \frac{x^3}{\sqrt{16-x^2}}dx$ trig sub



$\cos\theta = \dfrac{x}{4}$

$x = 4\cos\theta$

$\dfrac{dx}{d\theta} = -4\sin\theta$

$dx = -4\sin\theta\, d\theta$

$\sin\theta = \dfrac{\sqrt{16-x^2}}{4}$

$\sqrt{16-x^2} = 4\sin\theta$

$\displaystyle\int \frac{(4\cos\theta)^3}{4\sin\theta}(-4)\sin\theta\, d\theta$

$-64\displaystyle\int \cos^3\theta\, d\theta$

$-64\displaystyle\int \cos^2\theta \cos\theta\, d\theta$

$-64\displaystyle\int (1-\sin^2\theta)\cos\theta\, d\theta$

$u = \sin\theta$

$\dfrac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

$-64\displaystyle\int (1-u^2)\,du$

$-64u + \dfrac{64}{3}u^3 + C$

$-64\sin\theta + \dfrac{64}{3}\sin^3\theta + C$

$\boxed{-64\left(\dfrac{\sqrt{16-x^2}}{4}\right) + \dfrac{64}{3}\left(\dfrac{\sqrt{16-x^2}}{4}\right)^3 + C}$

#29 $\int \sin^2 x\,dx$

$\int \left(\frac{1-\cos(2x)}{2}\right) dx$

$\int \frac{1}{2}dx - \frac{1}{2}\int \cos(2x)dx$

$\boxed{\frac{1}{2}x - \frac{1}{4}\sin(2x) + C}$

#30 $\int \cos^2 x\,dx$

$\int \left(\frac{1+\cos(2x)}{2}\right)dx$

$\int \frac{1}{2}dx + \frac{1}{2}\int \cos(2x)dx$

$\boxed{\frac{1}{2}x + \frac{1}{4}\sin(2x) + C}$

#31 $\int \frac{1}{x^2-5x-14}$

$x^2-5x-14$

$(x-7)(x+2)$

factorable → partial fractions

$\frac{1}{(x-7)(x+2)} = \frac{A}{x-7} + \frac{B}{x+2}$

$A(x+2) + B(x-7) = 1$

$Ax+2A+Bx-7B = 1$

$(A+B)x + (2A-7B) = (0)x + (1)$

$\begin{cases} A+B=0 \\ 2A-7B=1 \end{cases}$

$-2A-2B=0$

$2A-7B=1$

$-9B=1$

$B=-1/9$

$A+B=0$

$A=1/9$

$\frac{1}{9}\int \frac{1}{x-7}dx - \frac{1}{9}\int \frac{1}{x+2}dx$

$\boxed{\frac{1}{9}\ln|x-7| - \frac{1}{9}\ln|x+2| + C}$

#32 $\int \frac{1}{x^2-12x+38}$

doesn't factor, so complete the square to try for arctan form:

$x^2-12x+\underline{\underline{36}}+38-\underline{\underline{36}}$

$(x-6)^2+2$ addition, so arctan will work :)

$\int \frac{1}{(x-6)^2+2}\,dx$

$u=x-6 \quad a=\sqrt{2}$

$\frac{du}{dx}=1$

$du=dx$

$\int \frac{1}{u^2+a^2}\,du$

$=\frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$\boxed{\frac{1}{\sqrt{2}}\arctan\left(\frac{x-6}{\sqrt{2}}\right)+C}$

#33 $\int \frac{1-\sin^2 x}{\cos x}\,dx$

$\sin^2 x+\cos^2 x=1$

so $1-\sin^2 x=\cos^2 x$

$\int \frac{\cos^2 x}{\cos x}\,dx$

$\int \cos x\,dx$

$\boxed{\sin x+C}$

#34 $\int xe^x\,dx$

by parts

$u=x \quad dv=e^x\,dx$

$\frac{du}{dx}=1 \quad \int dv=\int e^x\,dx$

$du=dx \quad v=e^x$

$uv-\int v\,du$

$xe^x-\int e^x\,dx$

$\boxed{xe^x-e^x+C}$

#35 $\int \csc^4 x \cot^3 x \, dx$

$u = \cot x$

$\frac{du}{dx} = -\csc^2 x \, dx$

$du = -\csc^2 x \, dx$

$\csc^2 x \, dx = -du$

$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\cot^2 x + 1 = \csc^2 x$

$\int \csc^2 x \cot^3 x \underline{\csc^2 x \, dx}$

$\int (\cot^2 x + 1) \cot^3 x \csc^2 x \, dx$

$\int (u^2+1) u^3 (-du)$

$-\int (u^5 + u^3) du$

$-\frac{1}{6}u^6 - \frac{1}{4}u^4 + C$

$\boxed{-\frac{1}{6}\cot^6 x - \frac{1}{4}\cot^4 x + C}$

#36 $\int_3^\infty x \ln(x^2) dx$

$\lim_{b\to\infty} \int_3^b x \ln(x^2) dx$

$\lim_{b\to\infty} \frac{1}{2}\left[x^2 \ln(x^2) - x^2\right]_3^b$

$\lim_{b\to\infty} \frac{1}{2}\left[b^2 \ln(b^2) - b^2\right] - \frac{1}{2}\left[3^2 \ln(3^2) - (3)^2\right]$

∞

$\boxed{\text{diverges}}$

$\int x \ln(x^2) dx$

u-sub

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x \, dx$

$x \, dx = \frac{1}{2} du$

$\frac{1}{2}\int \ln(u) du$

now, by parts so let's change letter first

$\frac{1}{2}\left[\int \ln(y) dy\right]$

$u = \ln(y)$ $\quad dv = dy$

$\frac{du}{dy} = \frac{1}{y}$ $\quad \int dv = \int dy$

$du = \frac{1}{y} dy$ $\quad v = y$

$\frac{1}{2}[uv - \int v \, du]$

$\frac{1}{2}[y \ln(y) - \int y \frac{1}{y} dy]$

$\frac{1}{2}[y \ln(y) - \int 1 \, dy]$

$\frac{1}{2}[y \ln(y) - y]$

$y = u = x^2$

back substitute

$\frac{1}{2}[x^2 \ln(x^2) - x^2]$

#37 $\int \frac{\sec x}{\tan^2 x+1}\,dx$

$$\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}$$

$$1+\tan^2 x=\sec^2 x$$

$\int \frac{\sec x}{\sec^2 x}\,dx$

$\int \frac{1}{\sec x}\,dx$

$\int \cos x\,dx$

$\boxed{\sin x + C}$

#38 $\int 7e^{5x}\,dx$

$\boxed{7\frac{e^{5x}}{5}+C}$

#39 $\int \cot x\,dx$

u-sub

$\int \frac{\cos x}{\sin x}\,dx$ $\quad u=\sin x$

$\frac{du}{dx}=\cos x$

$\int \frac{1}{u}\,du$ $\quad du=\cos x\,dx$

$\ln|u|+C$

$\boxed{\ln|\sin x|+C}$

#40 $\int 2x^3 \cos(x^2)\,dx$

u-sub

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\int x^2 \cos(x^2)\, 2x\,dx$

$\int u \cos(u)\, du$

now, by parts

(change letter first : $y = u$)

$\int y \cos(y)\,dy$

$u = y \qquad dv = \cos y\, dy$

$\frac{du}{dy} = 1 \qquad \int dv = \int \cos y\, dy$

$du = dy \qquad v = \sin y$

$uv - \int v\,du$

$y \sin y - \int \sin y\, dy$

$y \sin y + \cos y + C \qquad y = u = x^2$

$\boxed{x^2 \sin(x^2) + \cos(x^2) + C}$

(#41) $\int_0^\infty \frac{x^2}{(1-x^3)^2}dx$ two issues: $-\infty$ limit and
- vertical asymptote $x=1$ is with interval (must split)

$\lim_{b\to1^-}\int_0^b \frac{x^2}{(1-x^3)^2}dx + \lim_{\substack{d\to1^+\\ e\to\infty}}\int_d^e \frac{x^2}{(1-x^3)^2}dx$

$\int \frac{x^2}{(1-x^3)^2}dx$

uSub

$u=1-x^3$

$\frac{du}{dx}=-3x^2$

$du=-3x^2dx$

$x^2dx=-\frac{1}{3}du$

$\int \frac{1}{u^2}(-\frac{1}{3}du)$

$-\frac{1}{3}\int u^{-2}du$

$-\frac{1}{3}\frac{u^{-1}}{-1}$

$\frac{1}{3u} = \frac{1}{3(1-x^3)}$

$\lim_{b\to1^-}\left[\frac{1}{3(1-x^3)}\right]_0^b + \lim_{\substack{d\to1^+\\ e\to\infty}}\left[\frac{1}{3(1-x^3)}\right]_d^e$

$\lim_{b\to1^-}\left[\frac{1}{3(1-b^3)}\right) - \frac{1}{3(1-(0)^3)} + \lim_{e\to\infty}\left[\frac{1}{3(1-e^3)}\right] - \lim_{d\to1^+}\left[\frac{1}{3(1-d^3)}\right]$

$\infty \quad -\frac{1}{3} \quad 0 \quad -\infty$

$\boxed{\text{diverges}}$ (can stop as soon as you see any infinity)

#42 $\int 3\sec^2 x\,dx = \boxed{3\tan x + c}$

#43 $\int \frac{2t}{(t-3)^2}\,dt$

u-sub

$u = t-3$, $t = u+3$

$\frac{du}{dt} = 1$

$du = dt$

$\int \frac{2(u+3)}{u^2}\,du$

$\int \left(\frac{2u}{u^2} + \frac{6}{u^2}\right) du$

$2\int \frac{1}{u}\,du + 6\int u^{-2}\,du$

$2\ln|u| + 6\frac{u^{-1}}{-1} + C$

$\boxed{2\ln|t-3| - \frac{6}{t-3} + C}$

#44 $\int \tan x\,dx$

usub

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$\sin x\,dx = -du$

$\int \frac{\sin x}{\cos x}\,dx$

$\int \frac{1}{u}(-du)$

$-\ln|u| + c$

$\boxed{-\ln|\cos x| + c}$

#45 $\int \csc x \cot x \, dx = \boxed{-\csc x + C}$

#46 $\int x \ln(x) \, dx$

by parts

$u = \ln x$ $\qquad dv = x \, dx$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad \int dv = \int x \, dx$

$du = \frac{1}{x} dx$ $\qquad v = \frac{1}{2}x^2$

$uv - \int v \, du$

$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x^2 \frac{1}{x} dx$

$\frac{1}{2}x^2 \ln x - \frac{1}{2}\int x \, dx$

$\boxed{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}$

#47 $\int \frac{\sin x + \sec x}{\tan x} dx$

$\int \frac{\sin x}{\tan x} dx + \int \frac{\sec x}{\tan x} dx$

$\int \frac{\sin x}{\left(\frac{\sin x}{\cos x}\right)} dx + \int \frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x}\right)} dx$

$\int \frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} dx + \int \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} dx$

$\int \cos x \, dx + \int \frac{1}{\sin x} dx$

$\int \cos x \, dx + \int \csc x \, dx$

$\boxed{\sin x + \ln|\csc x - \cot x| + C}$

#48 $\int 3x \ln(x^2)\,dx$

u sub

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$\int 3 \ln(u)\left(\frac{1}{2}du\right)$

$\frac{3}{2}\int \ln(u)\,du$

← now, by parts (first, change letter)

$\frac{3}{2}\left[\int \ln(y)\,dy\right]$

$u = \ln y$ $\quad$ $dv = dy$

$\frac{du}{dy} = \frac{1}{y}$ $\quad$ $\int dv = \int dy$

$dy = \frac{1}{y}dy$ $\quad$ $v = y$

$\frac{3}{2}\left[uv - \int v\,du\right]$

$\frac{3}{2}\left[y\ln y - \int y \frac{1}{y}\,dy\right]$

$\frac{3}{2}\left[y\ln y - \int 1\,dy\right]$

$\frac{3}{2}\left[y\ln y - y\right]$ $\quad$ $y = u = x^2$

$$\boxed{\frac{3}{2}\left[x^2\ln(x^2) - x^2\right] + C}$$

#49 $\int \cos x(1+\sin^2 x)\,dx$

u sub

$u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$\int (1+u^2)\,du$

$u + \frac{1}{3}u^3 + C$

$\boxed{\sin x + \frac{1}{3}\sin^3 x + C}$

#50 $\int (x^3 + e^{4x} + \cos(x))\,dx$

$\boxed{\frac{1}{4}x^3 + \frac{e^{4x}}{4} + \sin(x) + C}$