AP Calculus BC – Unit 5, Part 1 Required Practice **Name:** answers

5.1 – Required Practice

#1. Find the area enclosed by $y=-x+1$ and $y=-x^2+3x+1$

10.667

#2. Find the area enclosed by $x=-y$ and $x=-y^2+2y$

4.5

#3. Find the area enclosed by $f(x) = \sqrt[3]{x-1}$ and $g = x-1$

$\frac{1}{2}$

On #4 and #5, sketch the region by hand (no calculator) and find the area enclosed by the curves (integrate by hand).

#4. $y = x^2 - 4x + 3$ and $y = -x^2 + 2x + 3$

$$\left[-\frac{2}{3}(3)^3 + 3(2)^2\right] - \left[-\frac{2}{3}(0)^3 + 3(0)^2\right]$$

On #4 and #5, sketch the region by hand (no calculator) and find the area enclosed by the curves (integrate by hand).

#5. $x = 4 - y^2$ and $x = y - 2$

$$\left[6(2) - \tfrac{1}{3}(2)^3 - \tfrac{1}{2}(2)^2\right] - \left[6(-3) - \tfrac{1}{3}(-3)^3 - \tfrac{1}{2}(-3)^2\right]$$

On the rest of this assignment, sketch the curves and find the area enclosed (use your calculator for the sketch and the integral evaluation).

#6. $y = 3x^3 - 3x$ and $y = 0$

1,5

#7. $y = x^2 - 1$, $y = -x + 2$, $x = 0$, and $x = 1$

2,167

#8. $x = y^2$ and $x = y + 2$

4,5

#9. $y = \sin(x)$ and $y = \cos(2x)$ $-\frac{\pi}{2} \le x \le \frac{\pi}{6}$

1,299

5.2 – Required Practice

#1. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y=x^2$, y-axis, $y=4$, in the first quadrant;
about the $y-axis$

25,133

#2. $y=x^2$, y-axis, $y=4$, in the first quadrant;
about the x-axis

80,425

#3. $y = x^2$, y-axis, $y = 4$, in the first quadrant; about $y = -2$

147.445

#4. $y = x^2$, $x = y^2$; about $x = -1$

3.037

#5. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

$y = 0, \quad y = \sin x, \quad 0 \le x \le \pi; \quad$ about $y = -2$

$$\int_0^{\pi} \pi(\sin(x)+2)^2\,dx - \int_0^{\pi} \pi(2)^2\,dx$$

#6. $y = x^2$, y-axis, $y = 4$, in the first quadrant; about y = - 3

180,956

Sketch and find the volume (use your calculator for the sketch and the integral evaluation).

#7. $y = 3x + 5,\ x = 2,\ x = 7,\ y = 0\quad \textit{about the x-axis}$

5670.575

#8. $y = x^2 + 4,\ x = 0,\ y = 8\quad \textit{about the y-axis}$

25.133

#9. $y = x^2, \; y = \sqrt{x}$ *about the* $x-$*axis*

0.942

#10. $y = 2x + 3, \; x = 0, \; y = 9$ *around* $y = 9$

113.097

#11. $y = -2x + 8, \ \ y = 0, \ \ x = 0$ *around* $x = 5$

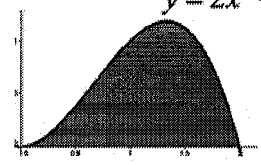
368.614

#12. $y = -2x + 8, \ \ y = 0, \ \ x = 0$ *around* $y = 9$

636.696

5.3 – Required Practice

#1. Find volume of solid obtained by rotating about the y-axis



10,053

#2. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. Sketch the region and a typical shell.

$$y = x^2 - 6x + 10, \quad y = -x^2 + 6x - 6$$

50.265

#3. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

$x = \sqrt{y}, \quad x = 0, \quad y = 1$

2.513

#4. Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

$y = x^2, \quad y = 0, \quad x = 1, \quad x = 2;$ about $x = 4$

35.081

#5. Set up, but **do not evaluate**, an integral....

$$x=\sqrt{\sin y},\quad 0\le y\le \pi,\quad x=0;\qquad \textbf{about } y=4$$

$$\int_0^{\pi} 2\pi(4-y)(\sqrt{\sin y})\,dy$$

#6. Use a graph to estimate the x-coordinates of the points of intersection of the given curves. Then use this information to estimate the volume of the solid obtained by rotating about the y-axis the region enclosed by these curves.

$$y=x^4,\quad y=3x-x^3$$

4.622

Sketch and find the volume using shell method (use your calculator for the sketch and the integral evaluation).

#7. $y = 3x + 5,\quad x = 0,\quad x = 7,\quad y = 0\qquad \textit{around the } y-axis$

2924.823

#8. $y = x^2 + 4,\quad x = 0,\quad y = 7\qquad \textit{around the } x-axis$

126.240

#9. $y = 2x + 3, \quad x = 0, \quad y = 9 \quad around\ y = 9$

113.097

#10. $y = x^2 - 4x + 9, \quad y = 2x + 1 \quad around\ x = 1$

16.255

#11. $y = \frac{10}{x^2}$, $y = 0$, $x = 1$, $x = 5$ *around the y – axis*

a) using Disk method...

101,124

b) using Shell method...

101,124

#12. $y=\frac{1}{x}, \quad y=0, \quad x=1, \quad x=2 \quad$ *around the x – axis*

a) using Disk method...

1,571

b) using Shell method...

1,571

Unit 5 Part 1 Test Review

For #1-4, find the area bounded by the given curves. **Sketch and setup the integral, but do not evaluate the integral.**

#1) $y = x^3, \quad y = x^2 - 4x + 4, \quad x = 2$

#2) $x - 2y + 7 = 0, \quad y^2 - 6y - x = 0$

#3) $y = e^{-x^2}, \quad y = 1 - \cos x, \quad x = 0$

#4) $y = 2^x, \quad y = 8, \quad x = 0$

For #5-8, use the **disk** method to find the volume generated by rotating the region bounded by the given curves about the specified axis. **Sketch and setup the integral, but do not evaluate the integral.**

#5) $y = x^2, \quad y = 4, \quad x = 0;$ *about the* $x-axis$

#6) $y = e^{-2x}, \quad y = 1 + x, \quad x = 1,$ *about the* $x-axis$

#7) $y = x^3, \quad y = 8, \quad x = 0,$ *about the* $y-axis$

#8) $y = x^3, \quad y = 8, \quad x = 0,$ *about* $x = 2$

For #9-14, use the **shell** method to find the volume generated by rotating the region bounded by the given curves about the specified axis. **Sketch and setup the integral, but do not evaluate the integral.**

#9) $y = x^2, \quad y = 0, \quad x = -2, \quad x = -1;$ *about the* $y-axis$

#10) $y = x^2, \quad y = 0, \quad x = 1, \quad x = 4;$ *about* $x = 4$

#11) $y = x^3, \quad y = x^2,$ *about* $y = 1$

#12) $x + 3 = 4y - y^2, \quad x = 0,$ *about the* $x-axis$

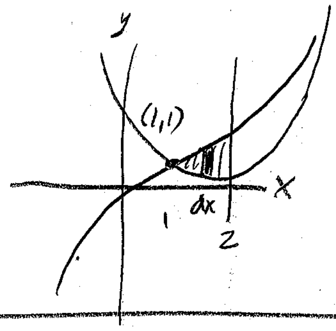
#13) $y = x^3, \quad y = 8, \quad x = 0,$ *about the* $y-axis$

#14) $y = \cos x, \quad y = 0, \quad x = \frac{3\pi}{2}, \quad x = \frac{5\pi}{2};$ *about the* $y-axis$

Unit 5 Part 1 Test Review — SOLUTIONS

#1

$$\boxed{A = \int_{1}^{2} \left[x^3 - (x^2 - 4x + 4)\right] dx}$$

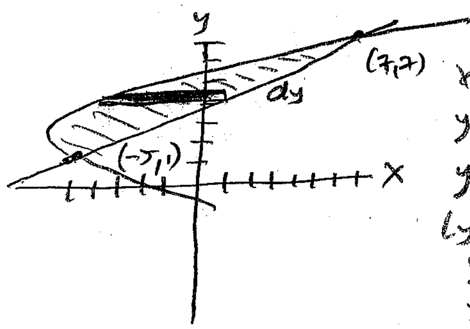


#2

$x = 2y - 7$

$x = y^2 - 6y$

$$\boxed{A = \int_{1}^{7} \left[(2y-7) - (y^2 - 6y)\right] dy}$$



Intersections: $\begin{cases} x - 2y + 7 = 0 \\ y^2 - 6y - x = 0 \end{cases}$

$x = 2y - 7$

$y^2 - 6y - (2y - 7) = 0$

$y^2 - 8y + 7 = 0$

$(y-7)(y-1) = 0$

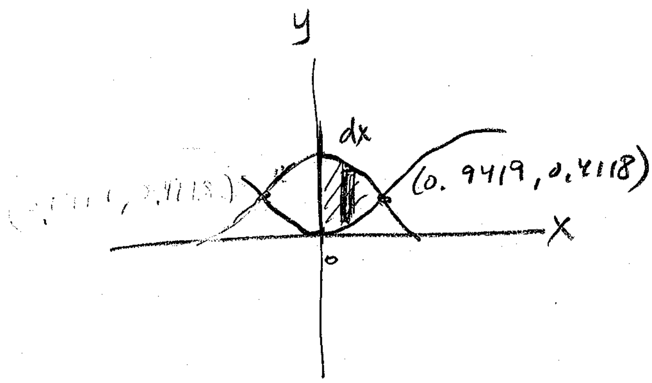
$y = 7 \quad y = 1$

$x = 7 \quad x = -5$

$(7,7) \quad (-5,1)$

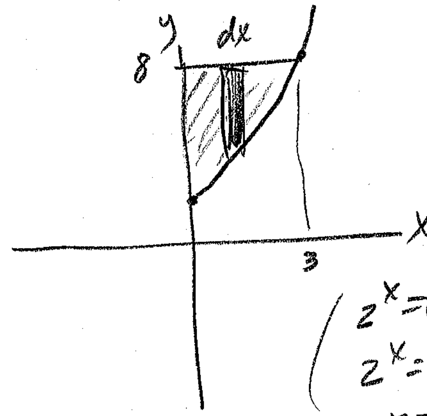
#3

$$\boxed{A = \int_{0}^{0.9419} \left[(e^{-x^2}) - (1 - \cos(x))\right] dx}$$



#4

$$\boxed{A = \int_0^3 [8 - 2^x]\, dx}$$

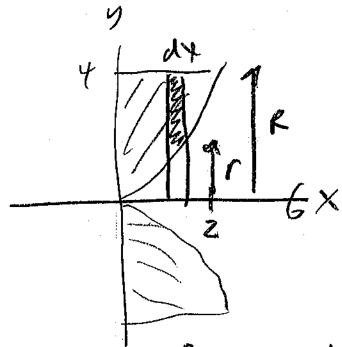


$$2^x = 8$$
$$2^x = 2^3$$
$$x = 3$$

#5

$$V = \int_a^b \pi R^2 dh - \int_a^b \pi r^2 dh$$

$$\boxed{\int_0^2 \pi (4)^2 dx - \int_0^2 \pi (x^2)^2 dx}$$

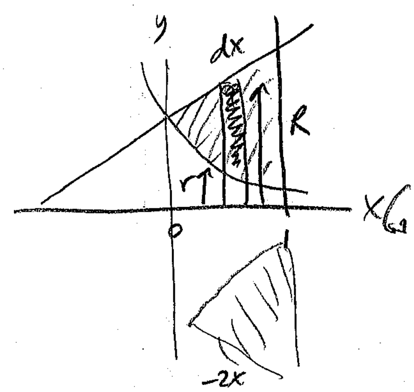


$r = x^2$, $R = 4$

#6

$$V = \int_a^b \pi R^2 dh - \int_a^b \pi r^2 dh$$

$$\boxed{\int_0^1 \pi (1+x)^2 dx - \int_0^1 \pi (e^{-2x})^2 dx}$$



$r = e^{-2x}$

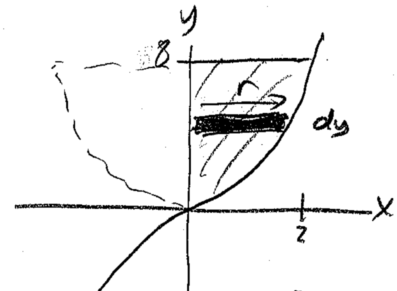
$R = 1 + x$

(#7) $y = x^3$

$x = \sqrt[3]{y}$

$$V = \int_a^b \pi r^2 \, dh$$

$$\boxed{\int_0^8 \pi (\sqrt[3]{y})^2 \, dy}$$



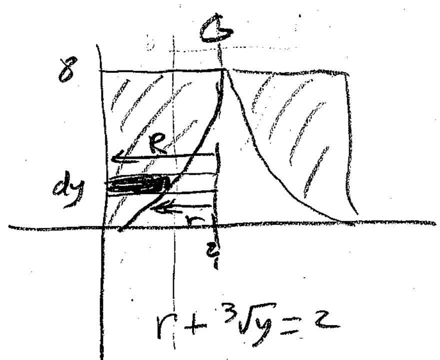
$r = \sqrt[3]{y}$

(#8) $y = x^3$

$x = \sqrt[3]{y}$

$$V = \int_a^b \pi R^2 \, dh - \int_a^b \pi r^2 \, dh$$

$$\boxed{\int_0^8 \pi (2)^2 \, dy - \int_0^8 \pi (2 - \sqrt[3]{y})^2 \, dy}$$



$r + \sqrt[3]{y} = 2$

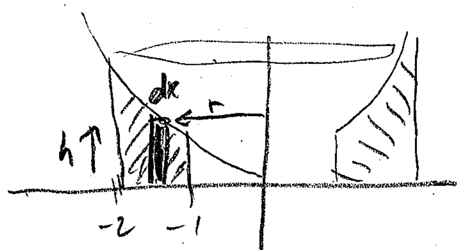
So $r = 2 - \sqrt[3]{y}$

$R = 2$

(#9)

$$V = \int_a^b 2\pi r h \, dr$$

$$\boxed{\int_{-2}^{-1} 2\pi (-x)(x^2) \, dx}$$



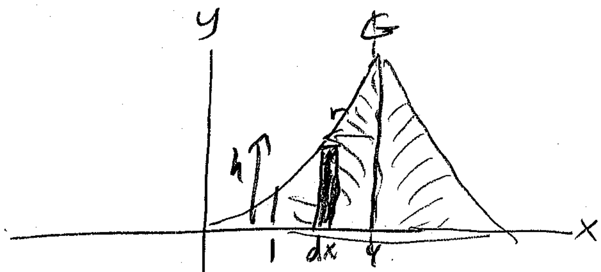
$h = x^2$

$r = -x$ (to give positive values)

#10 $y=x^2$

$$V=\int_a^b 2\pi r h\, dh$$

$$\boxed{\int_1^4 2\pi(4-x)(x^2)\,dx}$$



$h=x^2$

$r+x=4$

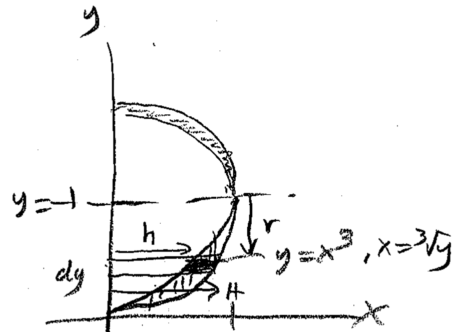
so $r=4-x$.

#11 $y=x^2$, $y=x^3$

$x=\sqrt{y}$ $\quad x=\sqrt[3]{y}$

$$V=\int_a^b 2\pi r H\, dr-\int_a^b 2\pi r h\, dr$$

$$\boxed{\int_0^1 2\pi(1-y)(\sqrt[3]{y})\,dy-\int_0^1 2\pi(1-y)(\sqrt{y})\,dy}$$



$H=\sqrt[3]{y}$

$h=\sqrt{y}$

$r+y=1$

so $r=1-y$

#12

$x+3=4y-y^2$

$x=4y-y^2-3$

Intersections: $\begin{cases} x+3=4y-y^2 \\ x=0 \end{cases}$

$0+3=4y-y^2$

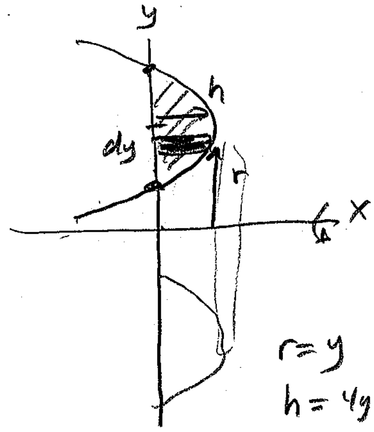
$y^2-4y+3=0$

$(y-1)(y-3)=0$

$y=1 \quad y=3$

$x=0 \quad x=0$

$(0,1) \quad (0,3)$



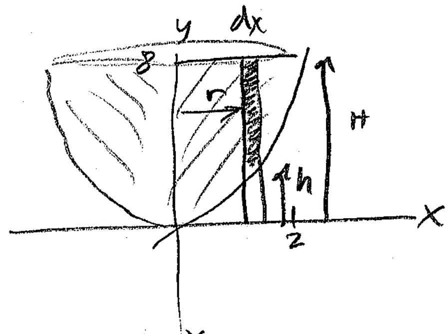
$r=y$

$h=4y-y^2-3$

$$V=\int_a^b 2\pi r h\,dr$$

$$\boxed{\int_0^3 2\pi(y)(4y-y^2-3)\,dy}$$

#13



$r=x$

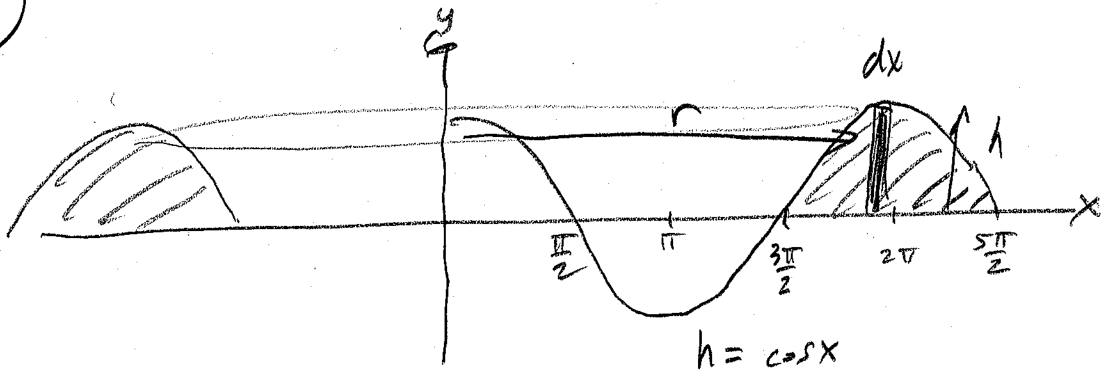
$h=x^3$

$H=8$

$$V=\int_a^b 2\pi r H\,dr-\int_a^b 2\pi r h\,dr$$

$$\boxed{\int_0^2 2\pi(x)(8)\,dx-\int_0^2 2\pi(x)(x^3)\,dx}$$

#14



$h = \cos x$

$r = x$

$$\boxed{V = \int_{3\pi/2}^{5\pi/2} 2\pi(x)(\cos(x))\,dx}$$