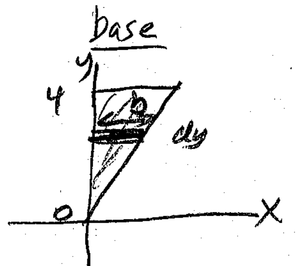
5.4 – Extra Practice

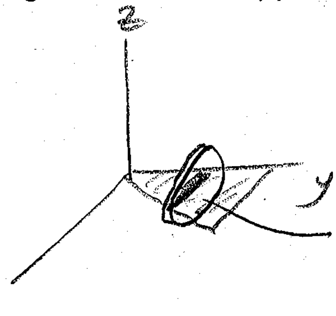
#5b. Find the volume of a shape that has cross-sections which are circles perpendicular to the y-axis, where the diameter of the circles are bounded by the triangular area on the x-y plane enclosed by $y = 2x, \quad y = 4, \quad x = 0$.

base



$y = 2x$

$x = \frac{y}{2}$



$b = \frac{y}{2}$

$$A_{cross} = \pi r^2 = \pi\left(\frac{b}{2}\right)^2 = \pi\left(\frac{\left(\frac{y}{2}\right)}{2}\right)^2 = \pi\left(\frac{y}{4}\right)^2 = \frac{\pi}{16}y^2$$

$$V = \int_a^b A_{cross}\,dy$$

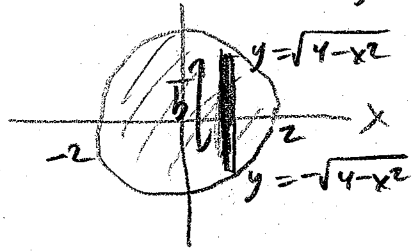
$$\boxed{\int_0^4 \frac{\pi}{16}y^2\,dy = 4.189}$$

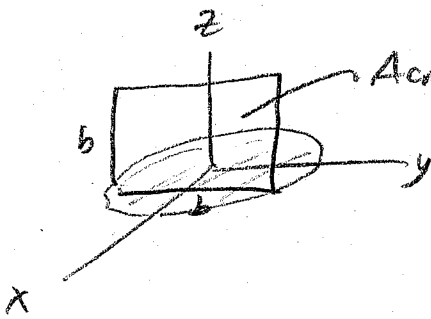
#6b. Find the volume of a shape that has cross-sections which are squares perpendicular to the x-axis, where the base of the object is bounded by $x^2 + y^2 = 4$.

base

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$





$$A_{cross} = b^2 = \left(2\sqrt{4-x^2}\right)^2$$

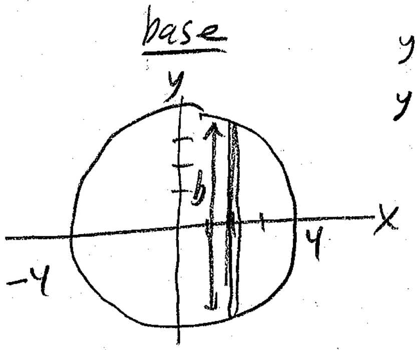
$b = \sqrt{4-x^2} - \left(-\sqrt{4-x^2}\right)$

$b = 2\sqrt{4-x^2}$

$$V = \int_a^b A_{cross}\,dx$$

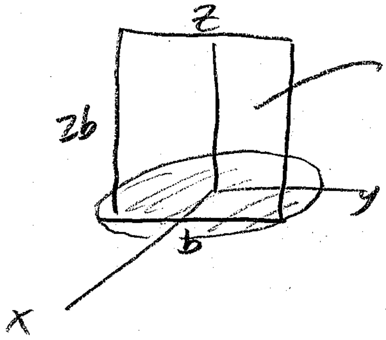
$$\boxed{\int_{-2}^{2} \left(2\sqrt{4-x^2}\right)^2 dx = 42.667}$$

#7b. Find the volume of a shape that has cross-sections which are rectangles with height twice the base, perpendicular to the *x*-axis, where the base of the object is bounded by $x^2+y^2=16$.



$y^2=16-x^2$

$y=\pm\sqrt{16-x^2}$



$A_{cross}=2b^2$

$=2\left(2\sqrt{16-x^2}\right)^2$

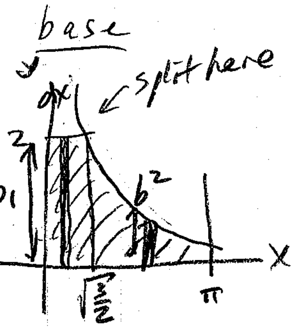
$b=\sqrt{16-x^2}-\left(-\sqrt{16-x^2}\right)$

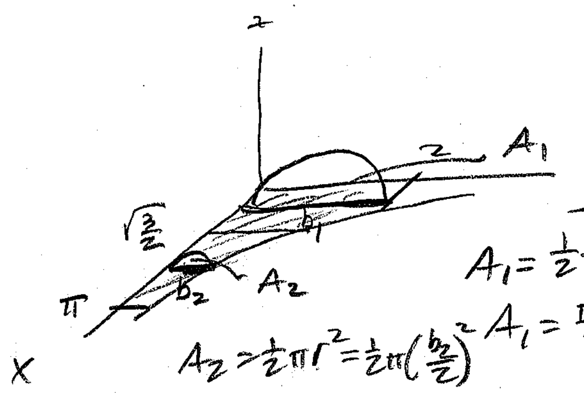
$b=2\sqrt{16-x^2}$

$V=\int_a^b A_{cross}\,dx$

$$\boxed{\int_{-4}^{4} 2\left(2\sqrt{16-x^2}\right)^2 dx = 682.667}$$

#8b. Let *R* be the region in the first quadrant bounded by the graph of $y=\dfrac{3}{x^2}$, the horizontal line $y=2$, and the vertical line $x=\pi$. Region *R* is the base of a solid which has semicircular cross sections perpendicular to the *x*-axis. Find the volume of the solid.





$2=\frac{3}{x^2}$

$x^2=\frac{3}{2}$

$x=\sqrt{\frac{3}{2}}$

$b_1=2$

$b_2=\frac{3}{x^2}$

$A_1=\frac{1}{2}\pi r^2=\frac{1}{2}\pi\left(\frac{b_1}{2}\right)^2$

$A_1=\frac{\pi}{2}\left(\frac{2}{2}\right)^2=\frac{\pi}{2}$

$A_2=\frac{1}{2}\pi r^2=\frac{1}{2}\pi\left(\frac{b_2}{2}\right)^2$

$A_2=\frac{\pi}{2}\left(\frac{\left(\frac{3}{x^2}\right)}{2}\right)^2=\frac{\pi}{2}\left(\frac{3}{2x^2}\right)^2$

$V=\int_0^{\sqrt{\frac{3}{2}}} A_1\,dx+\int_{\sqrt{\frac{3}{2}}}^{\pi} A_2\,dx$

$$\boxed{\int_0^{\sqrt{\frac{3}{2}}} \frac{\pi}{2}\,dx+\int_{\sqrt{\frac{3}{2}}}^{\pi} \frac{\pi}{2}\left(\frac{3}{2x^2}\right)^2 dx = 2.527}$$

$1.923824745+0.6032794712$

5.5 – Extra Practice

Find the arc length of the curve over the given interval.

#7b. $y = -2x^3 + x - 5 \qquad -1 \le x \le 5$

$y' = -6x^2 + 1$

$$\boxed{L = \int_{-1}^{5} \sqrt{1 + [-6x^2+1]^2}\, dx = 248.015}$$

#8b. $y = \ln(x) + 8 \qquad 3 \le x \le 7$

$y' = \frac{1}{x}$

$$\boxed{L = \int_{3}^{7} \sqrt{1 + [\tfrac{1}{x}]^2}\, dx = 4.094}$$

#9b. $x = y^3 - 2y^2 + 2 \qquad -1 \le y \le 3$

$x' = 3y^2 - 4y$

$$\boxed{L = \int_{-1}^{3} \sqrt{1 + [3y^2 - 4y]^2}\, dy = 15.603}$$

#10b. $x^2 + 2x + y^2 + 8y = 32 \qquad for\ x \ge 3$

$x^2 + 2x + 1 + y^2 + 8y \overset{+16}{=} 32 + 1 + 16$

$(x+1)^2 + (y+4)^2 = 49$

$(x+1)^2 + (y+4)^2 = 49$

$(x+1)^2 = 49 - (y+4)^2$

$x + 1 = \pm\sqrt{49 - (y+4)^2}$

here

$x = -1 \pm \sqrt{49 - (y+4)^2} = -1 + \sqrt{49 - (y+4)^2}$

$x' = \frac{1}{2}(49 - (y+4)^2)^{-1/2}(-2(y+4)(1))$

$$\boxed{L = \int_{-4-\sqrt{33}}^{+4+\sqrt{33}} \sqrt{1 + [\tfrac{1}{2}(49 - (y+4)^2)^{-1/2}(-2(y+4))]^2}\, dy = 13.476}$$

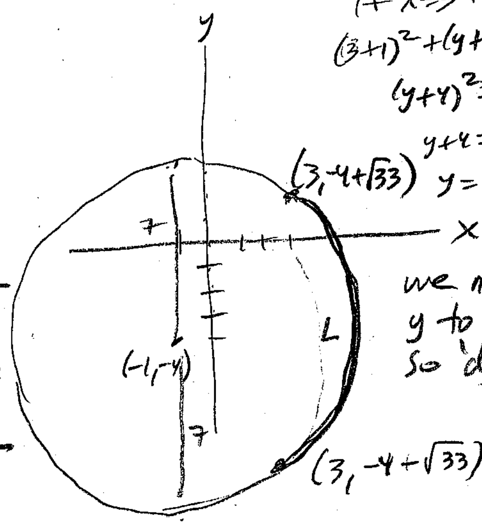
if $x = 3$:

$(3+1)^2 + (y+4)^2 = 49$

$(y+4)^2 = 33$

$y + 4 = \pm\sqrt{33}$

$y = -4 \pm \sqrt{33}$



we need y to change so 'dy' type

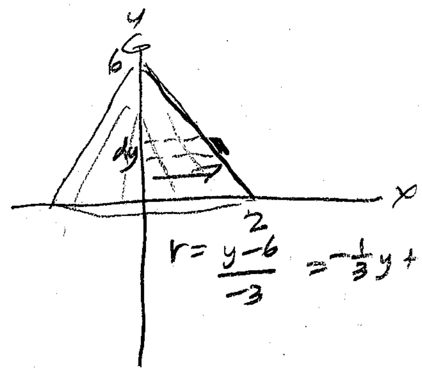
Find the surface area of the surface formed by rotating the portion of the curve indicated around the indicated axis.

#11b. *The portion of curve* $y=-3x+6$ *bounded by* $x=0,\ y=0$ *rotated around the* $y-axis$

$-3x = y-6$

$x = \frac{y-6}{-3} = -\frac{1}{3}y+2$

$x' = -\frac{1}{3}$



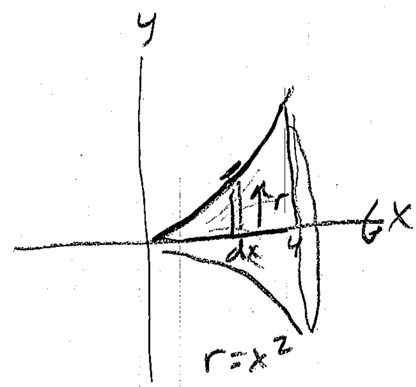
$r = \frac{y-6}{-3} = -\frac{1}{3}y+2$

$$A = \int_a^b 2\pi r\sqrt{1+[f'(y)]^2}\,dy$$

$$\boxed{\int_0^6 2\pi\left(-\tfrac{1}{3}y+2\right)\sqrt{1+\left(-\tfrac{1}{3}\right)^2}\,dy = 39{,}738}$$

#12b. *The portion of curve* $y=x^2$ *bounded by* $x=4,\ y=0$ *rotated around the* $x-axis$

$y' = 2x$



$r = x^2$

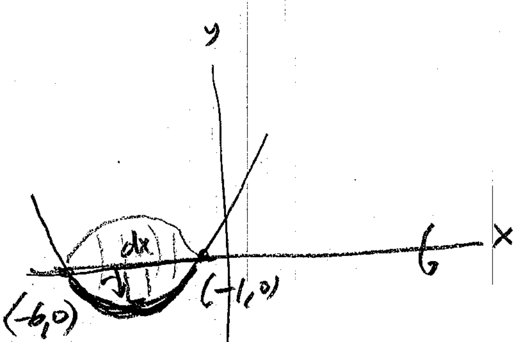
$$\boxed{A = \int_0^4 2\pi(x^2)\sqrt{1+[2x]^2}\,dx = 816.566}$$

#13b. *The portion of curve* $y-6=x^2+7x$ *bounded by* $y=0$ *rotated around the* $x-axis$

$y = x^2+7x+6$

$y' = 2x+7$

$$\boxed{A = \int_{-6}^{-1} 2\pi(-x^2-7x-6)\sqrt{1+[2x+7]^2}\,dx = 291{,}150}$$



$r = x^2+7x+6$ ← this is y which is negative but r must be positive so $r = -x^2-7x-6$

5.6 – Extra Practice

Find a) the average value of the function over the interval and
b) the average rate of change of the function over the interval

For this homework evaluate the integrals by hand

#6b. $y = 5x + 3$ $\quad 1 \le x \le 6$

a) avg value $= \frac{1}{6-1} \int_1^6 (5x+3)\,dx$

$= \frac{1}{5}\left[\frac{5}{2}x^2 + 3x\right]_1^6$

$\boxed{\frac{1}{5}\left(\left[\frac{5}{2}(6)^2 + 3(6)\right] - \left[\frac{5}{2}(1)^2 + 3(1)\right]\right)}$

b) avg rate of change $= \frac{y(6) - y(1)}{6-1}$

$= \boxed{\frac{(5(6)+3) - (5(1)+3)}{5}}$

#7b. $y = \frac{1}{x}$ $\quad 2 \le x \le 10$

a) avg value $= \frac{1}{10-2} \int_2^{10} \frac{1}{x}\,dx$

$= \frac{1}{8}\left[\ln|x|\right]_2^{10}$

$= \boxed{\frac{1}{8}\left(\ln(10) - \ln(2)\right)}$

b) avg rate of change $= \frac{y(10) - y(2)}{10-2}$

$= \boxed{\frac{\left(\frac{1}{10}\right) - \left(\frac{1}{2}\right)}{8}}$