AP Calculus BC – Unit 5, Part 2 Required Practice

Name: answers

5.4 – Required Practice

#1. Base bounded by $y = x^{1/2}$, $x = 9$, x-axis. Sections perpendicular to the x-axis are semicircles. Find the volume.

15.904

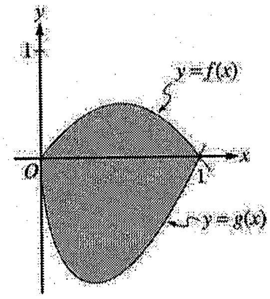
#2. Find the volume of the solid formed by cross-sections which are perpendicular to the x-axis and form squares. The base of the shape is in the x-y plane, defined by $y=x^2$, $y=0$, and $x=2$.

6.4

#3. Find the volume of the solid formed by cross-sections which are perpendicular to the x-axis and form equilaterial triangles where the base is one of the congruent sides. The base of the shape is in the x-y plane, defined by a rectangle which is 2 x 4.

13.856

#4.



Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1$. The graphs of f and g are shown in the figure above.

(a) Find the area of the shaded region enclosed by the graphs of f and g.

(b) Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line $y = 2$.

(c) Let h be the function given by $h(x) = kx(1-x)$ for $0 \le x \le 1$. For each $k > 0$, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

a) 1,133

b) 16,179

c) 15

#5. Find the volume of a shape that has cross-sections which are circles perpendicular to the *x*-axis, where the diameter of the circles are bounded by the triangular area on the x-y plane enclosed by $y=2x, \quad y=0, \quad x=4$.

67.021

#6. Find the volume of a shape that has cross-sections which are rectangles with height three times the base, perpendicular to the *x*-axis, where the base of the object is bounded by $y=2x, \quad y=0, \quad x=4$.

256

#7. Find the volume of a shape that has cross-sections which are squares perpendicular to the *y*-axis, where the base of the object is bounded by $x^2 + y^2 = 9$.

144

#8. Let *R* be the region in the first quadrant bounded by the graph of $y = \frac{1}{x}$, the horizontal line $y = 1$, and the vertical line $x = e$. Region *R* is the base of a solid which has semicircular cross sections perpendicular to the *x*-axis. Find the volume of the solid.

0,1376

5.5 – Required Practice

#1. Graph the curve and visually estimate its length. Then find its exact length.

$$y = \frac{x^3}{6} + \frac{1}{2x}, \quad \frac{1}{2} \le x \le 1$$

0.646

#2. Find the length of the curve.

$$y = \frac{x^4}{4} + \frac{1}{8x^2}, \quad 1 \le x \le 3$$

20.111

#3. Let *R* be the region enclosed by the graphs of $y = \ln(x^2 + 1)$ and $y = \cos(x)$.

What is the length of the boundary of region *R*?

4,282

#4. Find the area of the surface obtained by rotating the curve about the x-axis.

$$y = x^3, \ 0 \le x \le 2$$

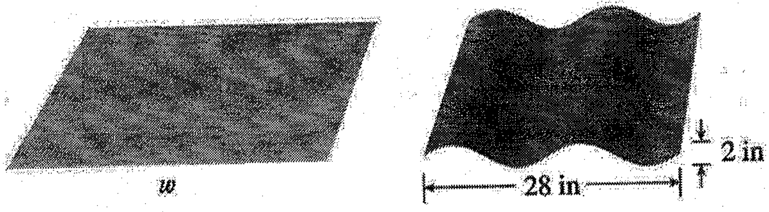
203,044

#5. The given curve is rotated about the y-axis. find the area of the resulting surface.

$$x = \sqrt{2y - y^2}, \quad 0 \le y \le 1$$

7,193

#6. A manufacturer of corrugated metal roofing wants to produce panels that are 28 in. wide and 2 in. thick by processing flat sheets of metal as shown in the figure. The profile of the roofing takes the shape of a sine wave. Verify that the sine curve has the equation $y = \sin(\pi x/7)$ and find the width w of a flat metal sheet that is needed to make a 28-inch panel.



29.361 in

Find the arc length of the curve over the given interval.

#7. $y = 3x^2 \qquad -3 \le x \le 3$

54.681

#8. $y = 4x^3 - 7x^2 + x - 27 \qquad -4 \le x \le 2$

382.734

#9. $x = y^2 + 3y - 4 \qquad 0 \le y \le 2$

10.209

#10. $x^2 - 4x + y^2 + 6y = 62 \qquad for\ x \ge 2$

27.207

Find the surface area of the surface formed by rotating the portion of the curve indicated around the indicated axis.

#11. *The portion of curve* $y=-3x+6$ *bounded by* $x=0,\ y=0$ *rotated around the* $x-$*axis*

119,215

#12. *The portion of curve* $y=x^2$ *bounded by* $x=0,\ y=9$ *rotated around the* $y-$*axis*

117,319

#13. *The portion of curve* $2x=y^2+6$ *bounded by* $y=0,\ y=2$ *rotated around the* $y-$*axis*

70,994

5.6 – Required Practice

#1. The number of people who've enrolled in an adult education class over the 40 day enrollment period is given by

$f(t)=\frac{1}{40}t^2+\frac{1}{2}t$ where t is in days and f is the number of people.

Find the average number of people enrolled over the enrollment period $0 \le t \le 40$

23.333 people

#2. The number of people who've enrolled in an adult education class over the 40 day enrollment period is given by

$f(t)=\frac{1}{40}t^2+\frac{1}{2}t$ where t is in days and f is the number of people.

Find the average rate of change in the number of people enrolled over the enrollment period $0 \le t \le 40$

1.5 $\frac{\text{people}}{\text{day}}$

#3. Find the average value of $f(x)=x^3-3x^2$ over the interval [1,4]

0.25

#4. Find the average rate of change of $f(x)=x^3-3x^2$ over the interval [1,4]

6

#5. An object's temperature in °C is changing over time and given by the function

$$T(t) = 25 + \frac{1}{10}t^2 + \cos(0.6t) \quad for \quad 0 \le t \le 20$$

where T is in °C and t is in hours.

a) What is the average temperature of the object over $0 \le t \le 20$?

b) What is the average rate of change of the object's temperature over $0 \le t \le 20$?

a) 38,289 °C

b) 1,992 °C/hr

Find a) the average value of the function over the interval and

b) the average rate of change of the function over the interval

For this homework evaluate the integrals by hand

#6. $y = x^2 + 3x - 2 \qquad -1 \le x \le 5$

a) $\frac{1}{6}\left(\left[\frac{1}{3}(5)^3 + \frac{3}{2}(5)^2 - 2(5)\right] - \left[\frac{1}{3}(1)^3 + \frac{3}{2}(1)^2 - 2(1)\right]\right)$

b) 7

#7. $y = 7x^3 + \sin(x) \qquad 0 \le x \le 1$

a) $\left(\frac{7}{4}(1)^4 - \cos(1)\right) - \left(\frac{7}{4}(0)^4 - \cos(0)\right)$

b) $\dfrac{\left[7(1)^3 + \sin(1)\right] - \left[7(0)^3 - \sin(0)\right]}{1}$

Unit 5 Part 2 Test Review

For #1-4, find the volume of the solid described. **Sketch and setup the integral, but do not evaluate the integral.**

#1) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$
and whose cross-sections are perpendicular to the y-axis and are squares.

#2) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$
and whose cross-sections are perpendicular to the y-axis and are semicircles. (assume base = diameter of the semicircle).

#3) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$
and whose cross-sections are perpendicular to the x-axis and are right, isosceles triangles with a leg in the base region.

For #5-6, find the average value of the function on the given interval. **No sketch is required, but set up <u>and</u> <u>evaluate</u> the integral (by hand).**

#4) $f(x) = -x^4 + 2x^2 + 4; \quad [-2, 1]$

#5) $f(x) = 4x^{\frac{1}{2}}; \quad [0, 3]$

For #6-7, find the length of the curve described. **No sketch required, setup the integral, but do not evaluate the integral.**

#6) $f(x) = 2(x-1)^{\frac{3}{2}}; \quad [1, 5]$

#7) $f(x) = \frac{x^3}{6} + \frac{1}{2x}; \quad [1, 3]$

For #8-11, find the surface area of the surface of revolution described. **Sketch and setup the integral, but do not evaluate the integral.**

#8) $y = \sin x, \quad 0 \le x \le \pi, \quad \textit{about the } x-\textit{axis}$

#9) $2y + x^2 = 1, \quad 0 \le x \le 1, \quad \textit{about the } y-\textit{axis}$

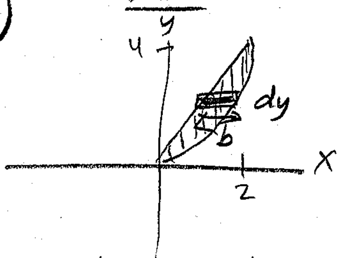
#10) $x - 1 = 2y^2, \quad 1 \le y \le 2, \quad \textit{about the } x-\textit{axis}$

#11) $x = \sqrt{2y - y^2}, \quad 0 \le y \le 1, \quad \textit{about the } y-\textit{axis}$

Unit 5 part 2 Test Review — SOLUTIONS

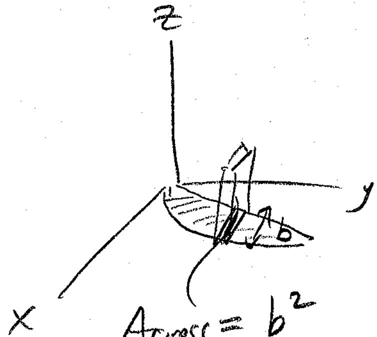
#1

base



$y = x^2 \qquad y = 2x$

$x = \sqrt{y} \qquad x = \frac{1}{2}y$



$b = \sqrt{y} - \frac{1}{2}y$

$A_{cross} = b^2$

$= (\sqrt{y} - \frac{1}{2}y)^2$

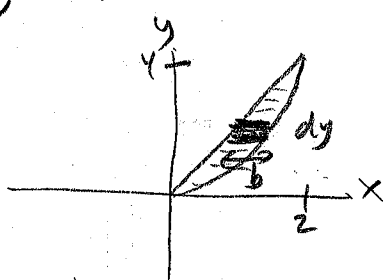
* 2D base sketch w/ rectangle required

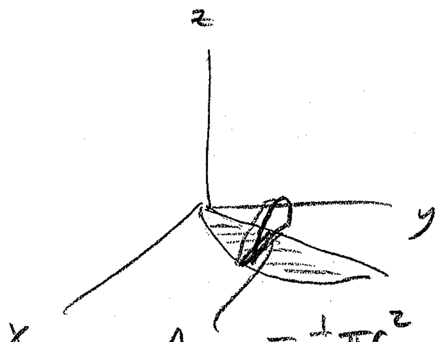
$V = \int_a^b A_{cross}\, dy = \boxed{\int_0^4 (\sqrt{y} - \tfrac{1}{2}y)^2\, dy}$

* 3D sketch not required

#2

base





$b = \sqrt{y} - \frac{1}{2}y$

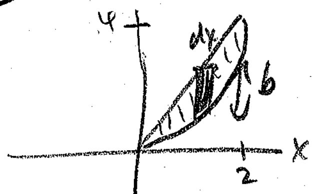
$A_{cross} = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\pi \left(\frac{b}{2}\right)^2$

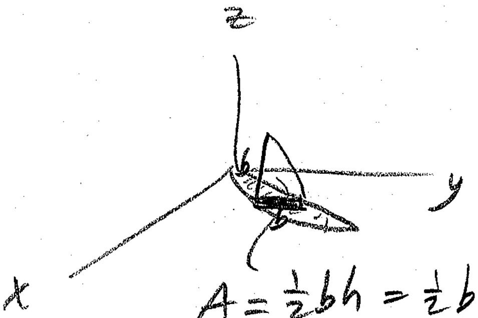
$= \frac{1}{2}\pi \left(\frac{\sqrt{y} - \frac{1}{2}y}{2}\right)^2$

$$\boxed{V = \int_0^4 \frac{1}{2}\pi \left(\frac{\sqrt{y} - \frac{1}{2}y}{2}\right)^2 dy}$$

#3 base



$b = 2x - x^2$



$A_{cross} = \frac{1}{2}bh = \frac{1}{2}b^2$

$A_{cross} = \frac{1}{2}(2x-x^2)^2$

$$\boxed{V = \int_0^2 \frac{1}{2}(2x-x^2)^2\,dx}$$

#4 Avg value $= \dfrac{1}{1-(-2)} \displaystyle\int_{-2}^{1} (-x^4+2x^2+4)\,dx$

$$\frac{1}{3}\left[-\frac{1}{5}x^5+\frac{2}{3}x^3+4x\right]_{-2}^{1}$$

$$\boxed{\frac{1}{3}\left(\left[-\frac{1}{5}(1)^5+\frac{2}{3}(1)^3+4(1)\right]-\left[-\frac{1}{5}(-2)^5+\frac{2}{3}(-2)^3+4(-2)\right]\right)}$$

#5 avg value $= \dfrac{1}{3-0}\displaystyle\int_0^3 4x^{1/2}\,dx$

$$\frac{1}{3}\cdot 4\left(\frac{2}{3}\right)\left[x^{3/2}\right]_0^3$$

$$\boxed{\frac{8}{9}\left[(3)^{3/2}-(0)^{3/2}\right]}$$

#6

$f(x) = 2(x-1)^{3/2}$

$f'(x) = 3(x-1)^{1/2}(1)$

$$\boxed{L = \int_1^5 \sqrt{1+[3(x-1)^{1/2}]^2}\,dx}$$

#7

$f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^{-1}$

$f'(x) = \frac{1}{2}x^2 - \frac{1}{2}x^{-2}$

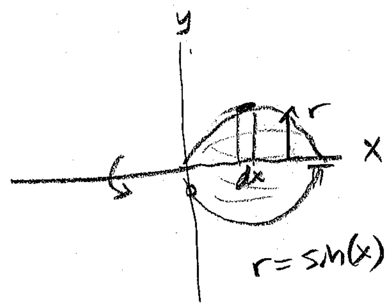
$$\boxed{L = \int_1^3 \sqrt{1+[\tfrac{1}{2}x^2 - \tfrac{1}{2}x^{-2}]^2}\,dx}$$

#8

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$$\boxed{A = \int_0^{\pi} 2\pi(\sin(x))\sqrt{1+[\cos(x)]^2}\,dx}$$



#9

$2y + x^2 = 1$

$2y = 1 - x^2$

$y = \frac{1}{2} - \frac{1}{2}x^2$

$x^2 = 1 - 2y$

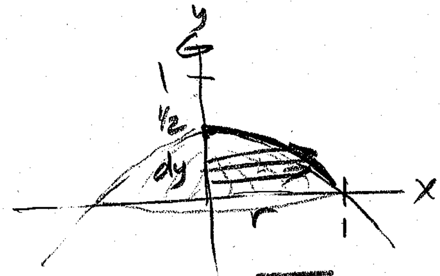
$x = \pm\sqrt{1-2y}$

here x is positive

$x = \sqrt{1-2y}$

$x = (1-2y)^{1/2}$

$x' = \frac{1}{2}(1-2y)^{-1/2}(-2)$



$r = \sqrt{1-2y}$

$$\boxed{A = \int_0^{1/2} 2\pi\left(\sqrt{1-2y}\right)\sqrt{1+\left[\frac{1}{2}(1-2y)^{-1/2}(-2)\right]^2}\,dy}$$

#10

$x - 1 = 2y^2$

$y = 1 \quad y = 2$

$x = 3 \quad x = 9$

$(3,1) \quad (9,2)$

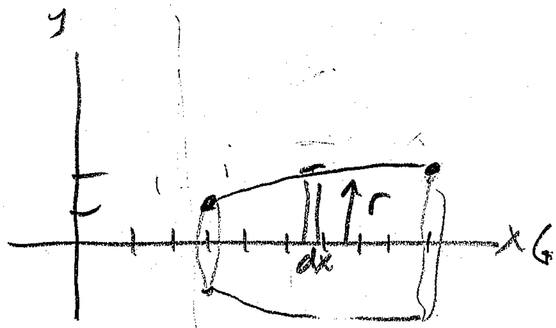
$y^2 = \frac{x-1}{2}$

$y = \pm\sqrt{\frac{x-1}{2}}$

here y is positive

$y = \sqrt{\frac{x-1}{2}} = \left(\frac{x-1}{2}\right)^{1/2}$

$y' = \frac{1}{2}\left(\frac{x-1}{2}\right)^{-1/2}\left(\frac{1}{2}\right)$



$r = \sqrt{\frac{x-1}{2}}$

$$\boxed{A = \int_3^9 2\pi\sqrt{\frac{x-1}{2}}\sqrt{1+\left[\frac{1}{2}\left(\frac{x-1}{2}\right)^{-1/2}\left(\frac{1}{2}\right)\right]^2}\,dx}$$

#11

$$x = (2y - y^2)^{1/2}$$

$$x' = \tfrac{1}{2}(2y - y^2)^{-1/2}(2 - 2y)$$

$$x = \sqrt{2y - y^2}$$

$$x^2 = 2y - y^2$$

$$x^2 + y^2 - 2y \overset{+1}{=} 0 + 1$$

$$x^2 + (y-1)^2 = 1$$

$$\boxed{A = \int_0^1 2\pi \sqrt{2y - y^2}\sqrt{1 + \left[\tfrac{1}{2}(2y - y^2)^{-1/2}(2 - 2y)\right]^2}\, dy}$$