AP Calculus BC – Unit 6 Required Practice **Name:** answers

6.1 – Required Practice

#1. Verify $x=-2$ is a solution to $x^2+x=2$

yes, $x=-2$ is a solution to $x^2+x=2$

#2. Verify $y=x^{-10}+3x^2$ is a solution to $x^2y''+9xy'=20y$

yes, $y=x^{-10}+3x^2$ is a solution to $x^2y''+9xy'=20y$

#3. Find the solution(s) for $\dfrac{dy}{dx}=0.1x\sqrt{x^2+1}$

$y=\frac{1}{30}(x^2+1)^{3/2}+C$

#4. For the differential equation $xy'' + y' = 0$

a) Verify that $y = C_1 + C_2 \ln(x)$ is the general solution to the differential equation.

b) Find the particular solution by using the initial conditions $y(2) = 0,\ y'(2) = \frac{1}{2}$

a) (verified)

b) $y = (-\ln(2)) + \ln(x)$

#5. Given the differential equation $y' = \frac{2}{x}$ and the initial condition $y(1) = 8$

a) Find the general solution

b) Use the initial condition to find the particular solution

c) Use the differential equation to sketch the first quadrant of a slope field for the differential equation. Include lineal elements for every 2 units in x and y from (2,2) to (10,10).

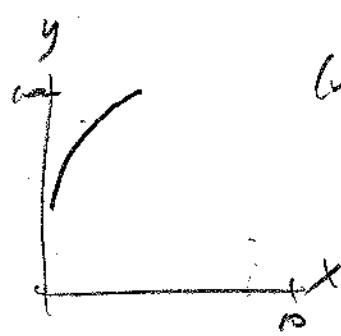
d) Use your calculator to graph your solution curve and add it to your slope field.

a) $y = 2\ln|x| + c$

b) $y = 2\ln|x| + 8$

c) (slope field graph)

d) solution curve roughly here



(w/ slope field lines)

Determine whether or not the given solution is a solution to the differential equation.

#6. *Differential Equation*: $y' = 5y$
Solution: $y = Ce^{5x}$

yes, $y = Ce^{5x}$ is a solution to $y' = 5y$

#7. *Differential Equation*: $2y + y' = 2\sin(2x) - 1$
Solution: $y = \sin(x)\cos(x) - \cos^2(x)$

yes, $y = \sin x \cos x - \cos^2 x$ is a solution to $2y + y' = 2\sin(2x) - 1$

#8. *Differential Equation*: $xy' - 2y = x^3 e^x$
Solution: $y = \ln(x)$

No, $y = \ln(x)$ is not a solution to $xy' - 2y = x^3 e^x$

#9. Verify the given solution is a solution to the differential equation. Then use the initial condition to find the particular solution.

Differential Equation: $y' + 6y = 0$

Solution: $y = Ce^{-6x}$

Initial condition: $y(0) = 3$

$y = 3e^{-6x}$

Use integration to find the general solution of the differential equation.

#10. $\dfrac{dy}{dx} = 12x^2$

$y = 4x^3 + C$

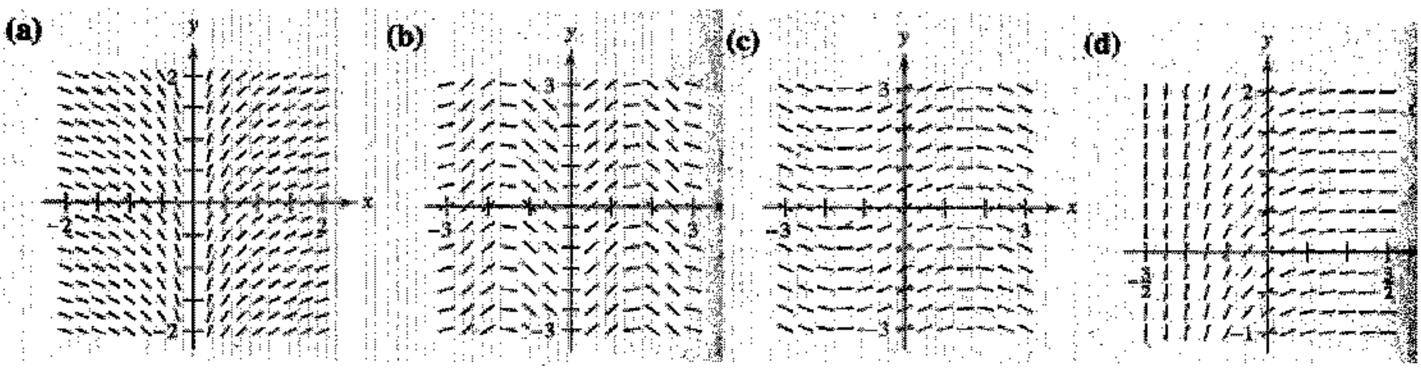
#11. $y' = \sin(2x)$

$y = -\frac{1}{2}\cos(2x) + C$

#12. Sketch a slope field for the differential equation $\frac{dy}{dx}=\frac{2x}{y}$. Include at least 6 points in the first quadrant.

(a slope field)

#13. Match the slope field with the differential equation:



$y' = \sin(2x)$ b

$y' = \frac{1}{2}\cos(x)$ c

$y' = e^{-2x}$ d

$y' = x^{-1}$ a

6.2 – Required Practice

#1. Use Euler's method to obtain a four-decimal approximation for $y(1.5)$

on the solution curve for $y' = 0.2xy \;\; with \;\; y(1) = 1$

Let's set $h = \Delta x = 0.1$, taking 5 iterations to reach from x=1 to x=1.5:

(x, y)	$y_{next} = y_{current} + f'(x_0, y_0)\,\Delta x$
	$y(1,5) \approx 1,126$

#2. The table gives values of $f'(x)$, the derivative of a function $f(x)$. If $f(1) = 4$ what is the approximation to $f(2)$ obtained by using Euler's method with a step size of 0.5?

x	$f'(x)$
1	0.2
1.5	0.5
2	0.9

$f(2) \approx 7,35$

#3. Use Euler's method to approximate $y(0.4)$ where $y(x)$ is the solution to $y' = x + y$ if $y(0) = 2$.
(use 4 equal size steps)

$y(0.4) \approx 2.992$

#4. The table gives values of $f'(x)$, the derivative of a function $f(x)$. If $f(2) = 5$
what is the approximation of $f(5)$ obtained by using Euler's method with a step size of 1?

x	$f'(x)$
2	0.5
3	0.3
4	0.2

$f(5) = 6.000$

6.3 – Required Practice

#1. $\dfrac{dy}{dx}=x^5$

$y=\frac{1}{6}x^6+C$

#2. $\dfrac{dy}{dx}=xy$

$y=Ce^{\frac{1}{2}x^2}$

#3. $2xy'-\ln(x)=0 \qquad y(1)=2$

$y=\frac{1}{4}(\ln(x))^2+2$

#4. $\frac{dy}{dx}=\frac{3x^2}{y^2}$

$y=\sqrt[3]{3x^3+c}$

#5. $\sqrt{x}+\sqrt{y}y'=0 \quad y(1)=9$

$y=(-x^{3/2}+28)^{2/3}$

Find the general solution of the differential equation. If an initial condition is given also find the particular solution.

#6. $\frac{dr}{ds} = .75r$

$r = C e^{0.75 s}$

#7. $yy' = 4\sin(x)$

$y^2 = -8\cos(x) + c$ implicit, general solution

or

$\left(y = \pm\sqrt{-8\cos(x) + C} \right)$ explicit, general solution

(you can leave in implicit form if solving for an explicit form would not result in a function)

#8. $\sqrt{1-4x^2}\,y' = x$

$y = -\frac{1}{4}\sqrt{1-4x^2} + C$

#9. $yy' - 2e^x = 0, \quad y(0) = 6$

$y^2 = 4e^x + C$

$\left(\text{or } y = \pm\sqrt{4e^x + C} \right)$

#10. $\frac{du}{dv} = uv\sin\left(v^2\right), \quad u(0)=1$

$$\ln|u| = -\tfrac{1}{2}\cos(v^2) + \tfrac{1}{2}$$

$$|u| = e^{\left(-\frac{1}{2}\cos(v^2)+\frac{1}{2}\right)}$$

$$u = \pm e^{\left(-\frac{1}{2}\cos(v^2)+\frac{1}{2}\right)}$$

(are all OK :-))

#11. $2xy' - \ln\left(x^2\right) = 0, \quad y(1) = 2$

$$y = \tfrac{1}{2}(\ln(x))^2 + 2$$

#12. A calf that weighs 60 pounds at birth gains weight at a rate given by $\frac{dw}{dt} = k(1200 - w)$

where w is in pounds and t in years.

a) Solve the differential equation.
b) Use a calculator to graph the particular solutions for $k = 0.8$, $k = 0.9$, and $k = 1$.
c) The animal is sold when its weight is 800 pounds. Find the time of sale for each of the models in part b.
d) What is the maximum weight of the animal for each of the models in part b?

a) $w = 1200 - 1140e^{-kt}$

b) a graph with curves approaching a horizontal asymptote

c) $k = .8$: $t = 1.309$ years
$k = .9$: $t = 1.164$ years
$k = 1$: $t = 1.047$ years

d) 1200 lbs

6.4 – Required Practice

#1. The population of bacteria increases at a rate which is proportional to the amount of bacteria. A culture initially has 200 bacteria. At $t = 1$ hr, the population of bacteria has increased to 300 bacteria. If the rate of growth is proportional to the number of bacteria present, determine the time needed for the bacteria population to quadruple.

3.419 hrs

#2. A murder victim's body is found by detectives who wish to establish the time of death. When the victim was alive, their body temperature was 98.6 °F, which begins cooling towards the ambient temperature as soon as death occurs. This victim was found in a building with air conditioning which maintained the ambient temperature at a constant 78°F. Detectives arrived on scene at 6:00am and found the core temperature of the body to be 84 °F. Core temperature was measured again at 6:30am and found to be 83 °F. What was the time of death?

3.383 hrs before 6:00AM (2:37 AM)

#3. Write and solve a differential equation for the statement "the rate of change of Q with respect to t is inversely proportional to the square of t."

$$\frac{dQ}{dt} = k\frac{1}{t^2}$$

$$Q = -\frac{k}{t} + C$$

#4. a) Write and solve a differential equation for the statement "the rate of change of N is proportional to N."
b) If $N = 250$ when $t = 0$ and $N = 400$ when $t = 1$ what is the value of N when $t = 4$?

a) $\frac{dN}{dt} = kN$, $N = Ce^{kt}$

b) 1638,400

#5. Radioactive element ^{229}Pu has a half-life of 24100 years. If the amount remaining after 10000 years is 0.4 grams, what was the initial quantity?

0.533 gram

#6. An investment has interest which compounds continuously with an annual interest rate of 12%. How long does it take for the amount invested to double?

5.776 years

6.5 – Required Practice

#1. The wolf population has unrestricted growth in a forest. There are 20 wolves at $t = 0$ months, and 40 wolves at $t = 10$ months. What will the wolf population be at time $t = 40$ months?

92,674 wolves

#2. The wolf population grows in a forest which can only support a maximum of 100 wolves. There are 20 wolves at $t = 0$ months, and 40 wolves at $t = 10$ months. What will the wolf population be at time $t = 40$ months?

320.004 wolves

#3. A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 200y - 0.2y^2$ where t is measured in years. What are all the values of y for which the population is increasing at a decreasing rate?

$500 < y < 1000$

#4. At time $t = 0$ a bacterial culture weighs 1 gram. Two hours later, the culture weighs 4 grams. The maximum weight of the culture is 20 grams.

a) Write a logistic differential equation that models the weight of the bacterial culture.

b) Solve the differential equation.

c) Find the culture's weight after 5 hours.

d) When will the culture's weight reach 18 grams?

e) After how many hours is the culture's weight increasing most rapidly?

a) $\frac{dP}{dt} = kP\left(1 - \frac{P}{20}\right) = 0.77907P\left(1 - \frac{P}{20}\right)$ (k came from part b)

b) $P(t) = \frac{20}{1 + 19e^{-0.77907t}}$

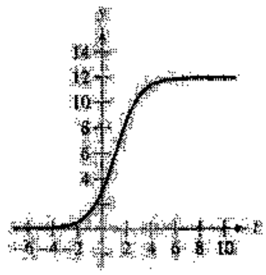
c) 14.426 grams

d) 6.600 hours

e) 3.778 hours

#5. Match the logistic equation with its graph.

A



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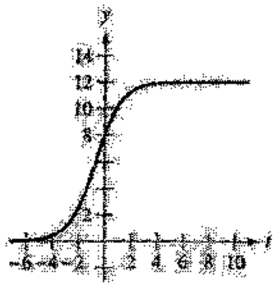
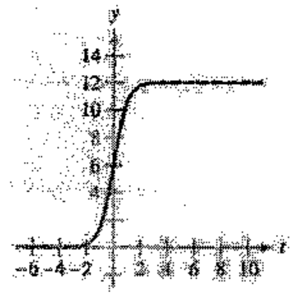
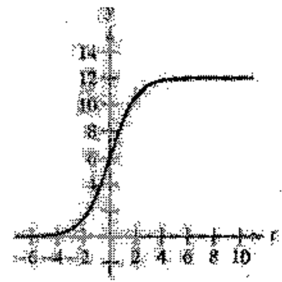


C



D



$y = \dfrac{12}{1+e^{-t}}$ D

$y = \dfrac{12}{1+3e^{-t}}$ A

$y = \dfrac{12}{1+\frac{1}{2}e^{-t}}$ B

$y = \dfrac{12}{1+e^{-2t}}$ C

#6. $P(t) = \dfrac{2100}{1+29e^{-0.75t}}$ models the growth of a population.

a) What is the carrying capacity and constant *k*, for this model?
b) Find the initial population.
c) Determine when the population will reach 50% of its carrying capacity.
d) Write the differential equation for which the given $P(t)$ is the solution.

a) $L = 2100$, $K = 0{,}75$

b) 70

c) 4,490 (years?)

d) $\frac{dP}{dt} = 0{,}75P\left(1 - \frac{P}{2100}\right)$

#7. $\frac{dP}{dt} = 3P - 0.03P^2$ models the growth of a population.

a) What is the carrying capacity and constant *k*, for this model?

b) Solve the differential equation given $P(0) = 10$

c) Determine the value of *P* at which the population growth rate is the greatest.

a) $L = 100$, $k = 3$

b) $P(t) = \frac{100}{1 + 9e^{-3t}}$

c) 50

#8. A conservation organization releases 25 Florida panthers into a game preserve. After 2 years, there are 39 panthers in the preserve. The Florida preserve has a carrying capacity of 200 panthers.

a) Write a logistic equation that models the population of panthers in the preserve.

b) Find the population after 5 years.

c) When will the population reach 100?

d) Write a logistic differential equation that models the growth rate of the panther population. Then repeat part b using Euler's method of approximation with a step size of 1. Compare the approximation with the exact number.

e) After how many years is the panther population growing most rapidly? Explain.

a) $P(t) = \frac{200}{1+7e^{-0,26403t}}$

b) 69,695 panthers

c) 7,370 years

d) $\frac{dP}{dt} = 0,264\, P\left(1-\frac{P}{200}\right)$, $P(5) \approx 65,559$ panthers

e) 7,370 years

Unit 6 Test Review

Solve the differential equation by separation of variables:

#1. $\frac{dy}{dx} = xy^2$ if $y(1) = 3$.

#2. $\frac{dy}{dx} = (2+x)y^2$ if $y(2) = 4$.

#3. $xyy' = 3 + x^2$ if $y(1) = 2$.

#4. $\frac{dP}{dt} = kP$ if $P(0) = 20$.

#5. $\frac{dy}{dt} = 0.5y - 0.004y^2$ if $y(0) = 30$.

Exponential Growth/Decay problems:

#6. When a child was born, her grandparents place \$1000 in a savings account which earns 12% annual interest compounded continuously.
(a) How much money is in the account when the child is 20 years old?
(b) At what time had the \$1000 doubled to \$2000?

#7. A radioactive sample which contained 50 g of mass at time $t = 0$ decays exponentially. After 75 days, the mass of the sample has decreased to 30 g.
(a) Write a differential equation which models this scenario.
(b) Solve the differential equation to write remaining mass as a function of time.
(c) What mass remains at $t = 100$ days?
(d) At what time was the initial mass reduced by 20%?

#8. A rabbit population with an initial size of 500 rabbits grows at a rate proportional to its size.
(a) Write a differential equation which models this scenario.
(b) Solve the differential equation to write the population of rabbits as a function of time, if there were 1200 rabbits at $t = 10$ days.
(c) How many rabbits will there be at $t = 50$ days ?
(d) When was the rabbit population 900 rabbits?

#9. A large manufacturing firm's training department has found that the number of minutes it takes assembly line workers to build a product decreases over time as the employees become more experienced, and that the time to build a product (in minutes) is inversely proportional to the number of days they have been building this product.
(a) Write a differential equation which models this scenario.
(b) Solve the differential equation to write the number of minutes to build a product as a function of days of experience, if it initially (in this case, the first day when t=1) takes employees 45 minutes to build the product, but they can build the product in 10 minutes when they have 30 days of experience.
(c) How many minutes does it take an employee to build the product after 10 days of experience?
(d) How many days of experience will it take for an employee to build the product in only 6 minutes?

Logistic equation problems:

#10. A virus spreads throughout a population according to the logistic differential equation $\frac{dy}{dt} = 0.5y - 0.004y^2$ where y is the number of people in the community who have been infected with the virus and t is the time in days.

(a) What is the size of this population?
(b) If 1 person is initially infected at $t = 0$, find the solution for the differential equation.
(c) At the time when the virus is spreading most quickly, how many people have caught the virus?
(d) To the nearest day, how many days will it take for half of the population to be infected?
(e) Draw and label a sketch of the graph of y as a function of time. (Label any significant parts)

#11. The rate of growth in a population of rabbits in a forest is proportional to current number of rabbits, but there is a limit in the food supply, so it cannot grow without bound. The carrying capacity of the forest is 40000 rabbits.

(a) What is the logistic form differential equation which models this situation?
(b) If there are 500 rabbits at $t = 0$, and 2000 rabbits at $t = 10$ days, find the solution for the differential equation.
(c) To the nearest day, how many days will it take for the population of rabbits to increase to 20000?
(d) At what time is the population of rabbits growing most rapidly?
(e) Draw and label a sketch of the graph of rabbit population as a function of time.

Euler's Method approximations:

#12. Given $\frac{dy}{dx} = 2x + 3y$ and $y(1) = 3$

use Euler's method to approximate $y(2)$ using increments of 0.2

Sketch slope fields:

#13. Sketch a slope field in quadrants 1 and 2 for the differential equation $\frac{dy}{dx} = -x + 2y$

(include lineal elements for at least 15 points in your sketch)

Unit 6 Test Review — SOLUTIONS

① $\frac{dy}{dx} = xy^2 \quad y(1) = 3$

$\frac{1}{y^2}dy = x\,dx$

$\int y^{-2}dy = \int x\,dx$

$\frac{y^{-1}}{-1} = \frac{1}{2}x^2 + c$

$-\frac{1}{y} = \frac{1}{2}x^2 + c$

$-\frac{1}{(3)} = \frac{1}{2}(1)^2 + c$

$\left(-\frac{1}{3} = \frac{1}{2} + c\right)6$

$-2 = 3 + 6c$

$6c = -5, \quad c = -5/6$

$-\frac{1}{y} = \frac{1}{2}x^2 - \frac{5}{6}$

$\boxed{y(x) = \frac{-1}{\left(\frac{1}{2}x^2 - \frac{5}{6}\right)}}$

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② $\frac{dy}{dx} = (2+x)y^2 \quad y(2) = 4$

$\frac{1}{y^2}dy = (2+x)dx$

$\int y^{-2}dy = \int (2+x)dx$

$\frac{y^{-1}}{-1} = 2x + \frac{1}{2}x^2 + C$

$-\frac{1}{y} = 2x + \frac{1}{2}x^2 + C$

$-\frac{1}{(4)} = 2(2) + \frac{1}{2}(2)^2 + C$

$-\frac{1}{4} = 4 + 2 + C$

$-\frac{1}{4} = 6 + C$

$-1 = 24 + 4C, \quad 4C = -25, \quad C = -\frac{25}{4}$

$-\frac{1}{y} = 2x + \frac{1}{2}x^2 - \frac{25}{4}$

$\boxed{y(x) = \frac{-1}{\left(2x + \frac{1}{2}x^2 - \frac{25}{4}\right)}}$

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③ $xy\underbrace{y'}_{dy/dx} = 3 + x^2 \quad y(1) = 2$

$y\frac{dy}{dx} = \frac{3+x^2}{x}$

$y\,dy = \left(\frac{3}{x} + x\right)dx$

$\int y\,dy = \int\left(\frac{3}{x} + x\right)dx$

$\frac{y^2}{2} = 3\ln|x| + \frac{1}{2}x^2 + C$

$\frac{(2)^2}{2} = 3\ln|1|^{0} + \frac{1}{2}(1)^2 + c$

$2 = \frac{1}{2} + c$

$4 = 1 + 2c$

$3 = 2c$

$c = \frac{3}{2}$

$\frac{y^2}{2} = 3\ln|x| + \frac{1}{2}x^2 + \frac{3}{2}$

$y^2 = 6\ln|x| + x^2 + 3$

$y = \pm\sqrt{6\ln|x| + x^2 + 3}$

$y(1) = 2$ is $+$

so

$\boxed{y = \sqrt{6\ln|x| + x^2 + 3}}$

(4) $\frac{dP}{dt} = kP \quad P(0)=20$

$\frac{1}{P}dP = k\,dt$

$\int \frac{1}{P}dP = \int k\,dt$

$\ln|P| = kt + C_1$

$\ln|20| = k(0) + C_1$

$C_1 = \ln|20|$

$\ln|P| = kt + \ln|20|$

$P = e^{(kt+\ln|20|)} = e^{kt} e^{\ln|20|} = e^{kt}(20)$

$\boxed{P = 20e^{kt}}$

the initial population

so $\boxed{P(t) = P_0 e^{kt}}$ in general (memorize this!)

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(5) $\frac{dy}{dt} = 0.5y - 0.004y^2 \quad y(0)=30$

$\frac{dy}{dt} = 0.5\left(y - \frac{.004}{0.5}y^2\right) = 0.5\left(y - \frac{1}{125}y^2\right)$

$\frac{1}{y - \frac{1}{125}y^2}dy = 0.5\,dt$

$\int \frac{1}{y(1-\frac{1}{125}y)}dy = \int 0.5\,dt \quad \text{--- (1)}$

partial fractions:

$\frac{1}{y(1-\frac{1}{125}y)} = \frac{A}{y} + \frac{B}{1-\frac{1}{125}y}$

$A(1 - \frac{1}{125}y) + By = 1$

$(-\frac{1}{125}A + B)y + A = (0)y + 1$

system: $\begin{cases} -\frac{1}{125}A + B = 0 \\ A = 1 \end{cases}$

$A = 1$

$B = \frac{1}{125}$

(1) $\int \frac{1}{y}dy + \left(\frac{1}{125}\right)\int \frac{1}{1-\frac{1}{125}y}dy = \int 0.5\,dt$

$u = 1 - \frac{1}{125}y$

$\frac{du}{dy} = -\frac{1}{125}$

$du = -\frac{1}{125}dy$

$dy = -125\,du$

$\ln|y| + \left(\frac{1}{125}\right)(-125)\int \frac{1}{u}du = 0.5t + C_1$

$\ln|1 - \frac{1}{125}y|$

log properties...

$\ln\left|\frac{y}{1-\frac{1}{125}y}\right| = 0.5t + C_1$

$\frac{y}{1-\frac{1}{125}y} = e^{(0.5t + C_1)} = C_2 e^{0.5t}$

$y = C_2 e^{0.5t}(1 - \frac{1}{125}y) = C_2 e^{0.5t} - C_2 e^{0.5}\frac{1}{125}y$

$y(1 + C_2 e^{0.5}\frac{1}{125}) = C_2 e^{0.5t}$

$y = \frac{C_2 e^{0.5t}}{1 + C_2 e^{0.5}\frac{1}{125}}$ $\longrightarrow$ on next page

5 continued

$$y = \frac{C_2 e^{0.5t}}{1 + C_2 e^{0.5t} \cdot \frac{1}{125}} = \frac{C_2 e^{0.5t} 125}{125 + C_2 e^{0.5t}} \left(\frac{e^{-0.5t}}{e^{-0.5t}} \right)$$

$$y = \frac{125 C_2}{125 e^{-0.5t} + C_2} \left(\frac{\frac{1}{C_2}}{\frac{1}{C_2}} \right)$$

$$y = \frac{125}{\frac{125}{C_2} e^{-0.5t} + 1}$$ define $C = \frac{125}{C_2}$ (one final constant)

$$\boxed{y = \frac{125}{1 + Ce^{-0.5t}}}$$ ← 125 is the "carrying capacity"

memorize these forms!

if you encounter $\frac{dy}{dt} = 0.5y - 0.004y^2$

rewrite: $\frac{dy}{dt} = 0.5y \left(1 - \frac{0.004}{0.5} y\right) = 0.5y \left(1 - \frac{y}{125}\right)$ ← carrying capacity

exponential constant

memorized solution form: $y(t) = \frac{125}{1 + Ce^{-0.5t}}$ ← (don't forget the negative

(6) $\frac{dA}{dt} = kA$

$\int \frac{1}{A} dA = \int k\, dt$

$\ln|A| = kt + C_1$

$A = e^{(kt+C_1)} = e^{kt} \underbrace{e^{C_1}}_{C} = Ce^{kt}$

$A = Pe^{rt}$ (P = principle)
(r = annual interest rate)

$A = Pe^{rt}$ ← (should memorize)

$A = 1000e^{.12t}$

(a) $A(20) = 1000e^{.12(20)} = \boxed{\$11023.18}$

(b) $2000 = 1000e^{.12t}$

$e^{.12t} = \frac{2000}{1000} = 2$

$.12t = \ln(2)$

$t = \frac{\ln(2)}{.12} = \boxed{5.77662 \text{ yrs}}$

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(7) radioactivity solution is also of form $Q = Q_0 e^{kt}$

(a) $\boxed{\frac{dQ}{dt} = kQ}$

(b) $Q = Q_0 e^{kt}$

$Q = 50e^{kt}$

$30 = 50e^{k(75)}$

$e^{75k} = \frac{30}{50}$

$75k = \ln\left(\frac{30}{50}\right)$

$k = \frac{\ln\left(\frac{30}{50}\right)}{75} = -0.0068110083$

$\boxed{Q = 50e^{-0.0068110083t}}$

(c) $Q(100) = 50e^{-0.0068110083(100)} = \boxed{25.30298 \text{ g}}$

(d) reduced by 20% means 80% remains $0.8(50)$

$0.8(50) = 50e^{-0.0068110083t}$

$e^{-0.0068110083t} = \frac{0.8(50)}{50} = 0.8$

$t = \frac{\ln(0.8)}{-0.0068110083} = \boxed{32.76219 \text{ days}}$

(8) (a) $\boxed{\frac{dP}{dt} = kP}$

(b) $P = P_0 e^{kt}$

$P = 500 e^{kt}$

$1200 = 500 e^{k(10)}$

$e^{10k} = \frac{1200}{500}$

$10k = \ln\left(\frac{1200}{500}\right)$ $\quad k = \frac{\ln\left(\frac{1200}{500}\right)}{10} = 0.0875468737$

$\boxed{P(t) = 500 e^{0.0875468737 t}}$

(c) $P(50) = 500 e^{0.0875468737(50)} = 3981.312$

$\boxed{3981 \text{ rabbits}}$

(d) $900 = 500 e^{0.0875468737 t}$

$t = \frac{\ln\left(\frac{900}{500}\right)}{0.0875468737} = \boxed{\cancel{33.01} \text{ days}}$

6.714

(9) (a) m = minutes to build

t = days experience

$\boxed{\frac{dm}{dt} = k \frac{1}{t}}$

(b) $\frac{1}{k} dm = \frac{1}{t} dt$

$\frac{1}{k} \int 1 \, dm = \int \frac{1}{t} dt$

$\frac{1}{k} m = \ln|t| + C$

$\frac{1}{k}(45) = \ln|1| + C$ $\quad C = \frac{45}{k}$

$\frac{1}{k} m = \ln|t| + \frac{45}{k}$

$m = k \ln|t| + 45$ $\quad$ now $m = 10$ when $t = 30$

$10 = k \ln|30| + 45$

$k \ln|30| = -35$

$k = \frac{-35}{\ln|30|} = -10.29049363$

$\boxed{m(t) = (-10.29049363) \ln|t| + 45}$

(c) $m(10) = -10.29049363 \ln|10| + 45$

$= \boxed{21.30526 \text{ minutes}}$

(d) $6 = -10.29049363 \ln|t| + 45$

$-10.29049363 \ln|t| = -39$

$\ln|t| = \frac{-39}{-10.29049363} = 3.789905655$

$t = e^{3.789905655} = \boxed{44.2522 \text{ days}}$

(10) $\frac{dy}{dt} = 0.5y - 0.004y^2$ (see #5)

$\frac{dy}{dt} = 0.5y\left(1 - \frac{y}{125}\right)$ carrying capacity = 125
exp. constant = 0.5

(a) size of population = carrying capacity = $\boxed{125}$

(b) solution (memorized form): $y(t) = \frac{125}{1+Ce^{-0.5t}}$ $y(0)=1$

$$1 = \frac{125}{1+Ce^{-0.5(0)}} = \frac{125}{1+C}$$

$$1+C = 125$$

$$C = 124$$

$$\boxed{y(t) = \frac{125}{1+124e^{-0.5t}}}$$

(c) logistic equations have max rate of increase at half carrying capacity (memorized fact)

So when $\frac{125}{2} = 62.5$ $\boxed{(62) \text{ people}}$

(d) $62.5 = \frac{125}{1+124e^{-0.5t}}$

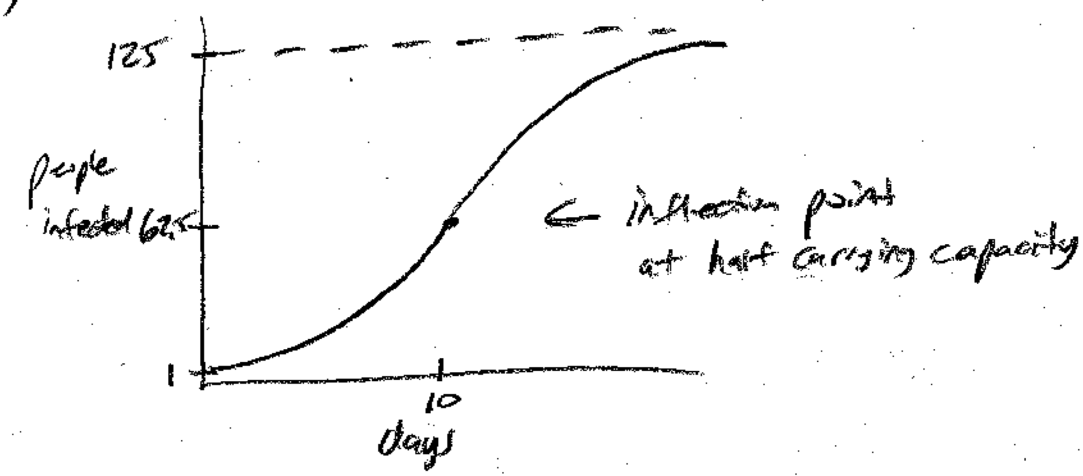
$1+124e^{-0.5t} = \frac{125}{62.5} = 2$

$124e^{-0.5t} = 1$

$e^{-0.5t} = \frac{1}{124}$

$-0.5t = \ln\left(\frac{1}{124}\right)$, $t = \frac{\ln\left(\frac{1}{124}\right)}{-0.5} = 9.164$ $\boxed{10 \text{ days}}$

(e)



(11) unlike #8 (unlimited growth, $P = P_0 e^{kt}$) now we have growth limited by environment, logistic growth

(a) carrying capacity $= 40000$

D.E. form: $\boxed{\frac{dP}{dt} = kP\left(1 - \frac{P}{40000}\right)}$

(b) Solution form: $P(t) = \frac{40000}{1 + Ce^{kt}}$ 2 data points:

t	P
0	500
10	2000

use $P(0) = 500$:

$500 = \frac{40000}{1 + Ce^{k(0)}} = \frac{40000}{1 + C}$

$500 + 500C = 40000$

$C = \frac{39500}{500} = 79$

$P = \frac{40000}{1 + 79e^{kt}}$

now use $P(10) = 2000$:

$2000 = \frac{40000}{1 + 79e^{k(10)}}$

$2000 + 2000(79)e^{10k} = 40000$

$2000(79)e^{10k} = 38000$

$e^{10k} = \frac{38000}{2000(79)} = \frac{19}{79}$

$10k = \ln\left(\frac{19}{79}\right)$, $k = \frac{\ln\left(\frac{19}{79}\right)}{10} = -0.1425008873$

$\boxed{P(t) = \frac{40000}{1 + 79e^{-0.1425008873t}}}$

(c) $20000 = \frac{40000}{1 + 79e^{-0.1425 \cdot t}}$

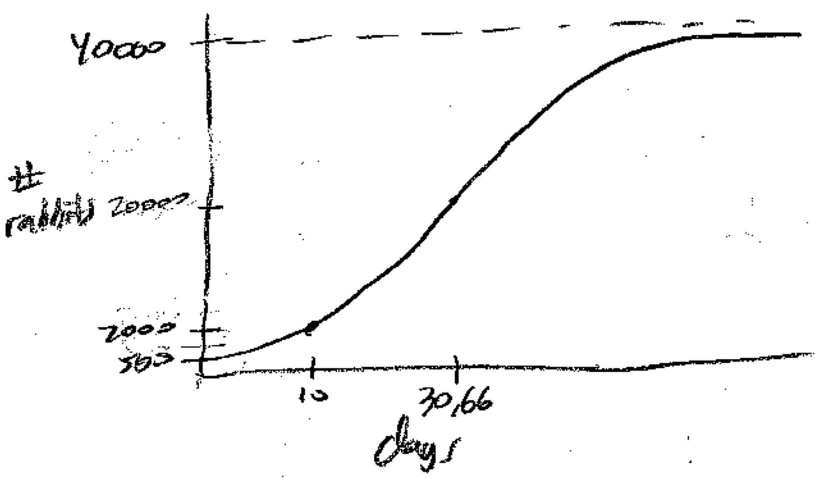
$1 + 79e^{-0.1425t} = \frac{40000}{20000} = 2$

$79e^{-0.1425t} = 1$

$t = \frac{\ln\left(\frac{1}{79}\right)}{-0.1425008873} = \boxed{30.6626 \text{ days}}$

(d) occurs when at half carrying capacity, which is 20000 so $\boxed{30.6626 \text{ days}}$

(e)



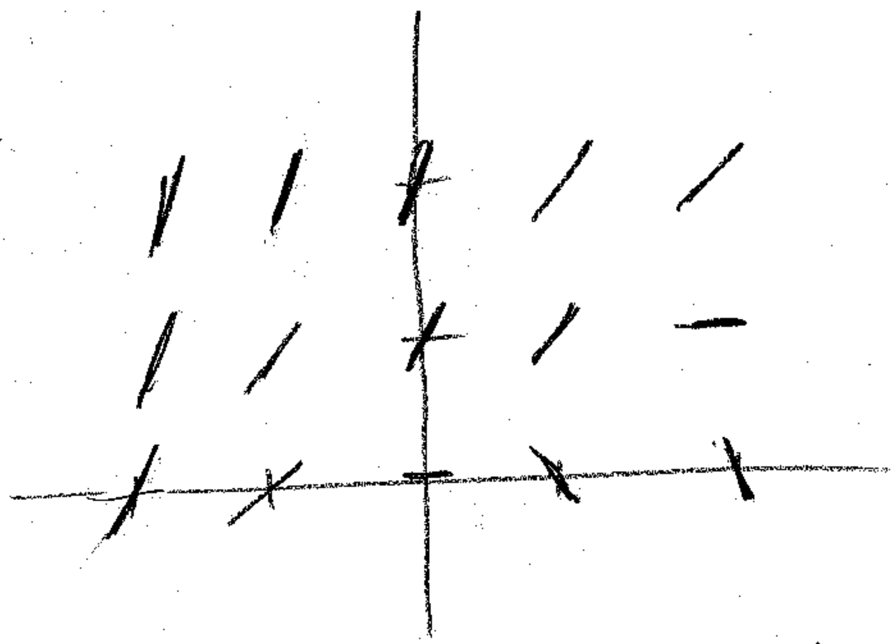
(12) $\frac{dy}{dx} = 2x + 3y \quad y(1) = 3$

x	y	$\Delta y = (2x+3y)\Delta x$
1	3	$(2(1)+3(3))(0.2) = 2.2$
1.2	5.2	$(2(1.2)+3(5.2))(0.2) = 3.6$
1.4	8.8	$(2(1.4)+3(8.8))(0.2) = 5.84$
1.6	14.64	$(2(1.6)+3(14.64))(0.2) = 9.424$
1.8	24.064	$(2(1.8)+3(24.064))(0.2) = 15.1584$
2	39.2224	

$$\boxed{y(2) \approx 39.2224}$$

(13) $\frac{dy}{dx} = -x + 2y$

(x,y)	$\frac{dy}{dx} = -x + 2y$
(0,0)	$-0 + 2(0) = 0$
(0,1)	$-0 + 2(1) = 2$
(0,2)	$-0 + 2(2) = 4$
(1,0)	$-1 + 2(0) = -1$
(1,1)	$-1 + 2(1) = 1$
(1,2)	$-1 + 2(2) = 3$
(2,0)	$-2 + 2(0) = -2$
(2,1)	$-2 + 2(1) = 0$
(2,2)	$-2 + 2(2) = 2$
(−1,0)	$-(-1) + 2(0) = 1$
(−1,1)	$-(-1) + 2(1) = 3$
(−1,2)	$-(-1) + 2(2) = 5$
(−2,0)	$-(-2) + 2(0) = 2$
(−2,1)	$-(-2) + 2(1) = 4$
(−2,2)	$-(-2) + 2(2) = 6$



(very rough sketch is fine ☺)