AP Calculus BC – Unit 7 Required Practice **Name: ___________________________________**

7.1 – Required Practice

#1. Write out the first 4 terms of the sequence defined by $a_n = (-1)^{n-1}\dfrac{n}{n+1}$

#2. $a_n = \dfrac{5n^2+3n}{2n^2-n+4}$

#3. $a_n = \dfrac{n!}{(n+1)!}$

#4. List the first 4 terms of the sequence:

$a_n = 1-(0.2)^n$

#5. Find an expression for the nth term:

$\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27} ....\right\}$

#6. Determine if the sequence converges and if so find the limiting value:

$a_n = \frac{3+5n^2}{n+n^2}$

$a_n = \frac{n!}{2^n}$

#7. Determine if the sequence is monotonic, bounded, and if it converges:

$a_n = \frac{n}{n^2+1}$

Write the first 5 terms of the sequence:

#8. $a_n = 4n-3$

#9. $a_n = (-1)^{n+1}\left(\frac{2}{n}\right)$

#10. Write the next two apparent terms

of the sequence: $2, 5, 8, 11, \ldots$

#11. Simplify: $\frac{(2n-1)!}{(2n+1)!}$

#12. Find the limit (if possible) of the sequence: $a_n = \frac{3n+1}{5n-2}$

Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find its limit:

#13. $a_n = \frac{5}{n+2}$

#14. $a_n = (-1)^n \left(\frac{n}{n+1}\right)$

#15. $a_n = \frac{(n+1)!}{n!}$

#16. $a_n = \frac{\sin(n)}{n}$

#17. Determine whether the sequence is monotonic and whether it is bounded.

$a_n = 4 - \frac{1}{n}$

#18. Find an expression for the nth term:

$\frac{1}{(2)(3)}, \frac{2}{(3)(4)}, \frac{3}{(4)(5)}, \frac{4}{(5)(6)}, \ldots$

7.2 – Required Practice

Examples: Determine if the series converges and if so to what sum.

#1. $\sum_{n=0}^{\infty} 3\left(\frac{1}{4}\right)^n$

#2. $\sum_{n=1}^{\infty} 3\left(\frac{1}{4}\right)^n$

#3. $\sum_{n=1}^{\infty} 3(1.1)^n$

#4. Write the repeating decimal as a geometric series and as a ratio of two integers: $0.\overline{36}$

Determine if the series converges and if so to what sum.

#5. $\sum_{n=1}^{\infty} \frac{n+1}{2n-1}$

#6. $\sum_{n=1}^{\infty} \frac{1}{n}$

Examples

Determine if the series converges and if so to what sum.

#7. $-2+\frac{5}{2}-\frac{25}{8}+\frac{125}{32}-...$

#8. $\sum_{n=1}^{\infty}\ln\left(\frac{n}{2n+5}\right)$

Determine if the series converges and if so to what sum.

#9. $\sum_{n=1}^{\infty}\frac{n}{\sqrt{1+n^2}}$

Find the values of x for which the series converges, and find the sum of the series for those values of x.

#10. $\sum_{n=1}^{\infty}\frac{x^n}{3^n}$

Find the sequence of partial sums S_1, S_2, S_3, and S_4.

#11. $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\dots$

#12. $3-\frac{9}{2}+\frac{27}{4}-\frac{81}{8}+\frac{243}{16}-\dots$

Verify that the infinite series diverges:

#13. $\sum_{n=0}^{\infty} 5\left(\frac{5}{2}\right)^{n}$

#14. $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$

Verify that the infinite series converges:

#15. $\sum_{n=0}^{\infty}\left(\frac{5}{6}\right)^n$

#16. $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

#17. Find the sum of the convergent series

$$\sum_{n=1}^{\infty}-\left(\frac{1}{5}\right)^n$$

#18. Write the repeating decimal as a geometric
And write the sum of the series as a fraction:

$$0.\overline{81}$$

Determine if the series is convergent or divergent:

#19. $\sum_{n=0}^{\infty} \frac{3^n}{1000}$

#20. $\sum_{n=2}^{\infty} \frac{n}{\ln(n)}$

#21. $\sum_{n=1}^{\infty} e^{-n}$

7.3 – Required Practice

Use the Integral Test to determine if the series converges or diverges:

#1. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

#2. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

Use the Integral Test to determine if the series converges or diverges:

#3. $\sum_{n=3}^{\infty} \frac{1}{n^2 + 1}$

Determine if the series converges or diverges:

#4. $\sum_{n=1}^{\infty} \frac{1}{n}$

#5. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Determine if the series converges or diverges:

#6. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

#7. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

#8. $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

Confirm that the integral test applies, then use it to determine if the series converges or diverges.

#9. $\sum_{n=1}^{\infty} \frac{1}{n+3}$

#10. $\sum_{n=1}^{\infty} e^{-n}$

Confirm that the integral test applies, then use it to determine if the series converges or diverges.

#11. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

#12. $\sum_{n=1}^{\infty} \frac{4n}{2n^2 + 1}$

Explain why the integral test does not apply to the series.

#13. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

#14. $\sum_{n=1}^{\infty} \frac{2+\sin(n)}{n}$

Use the p-series test to determine the convergence or divergence of the series.

#15. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

#16. $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$

#17. $\sum_{n=1}^{\infty} \frac{1}{n^{1.03}}$

7.4 – Required Practice

#1. Example: Determine if the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{3^n}{2^n - 1}$

#2. Example: Determine if the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{1}{3n^2 - 4n + 5}$

#3. Example: Determine if the following series converges or diverges: $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

#4. Example: Determine if the following series converges or diverges: $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$

(use limit comparison test)

Determine if the following series converge or diverge

#5. $\sum_{n=1}^{\infty} \frac{1}{n^3 - 1}$

#6. $\sum_{n=1}^{\infty} \frac{5}{2 + 3^n}$

#7. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$

#9. $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$ (use Limit comparison)

#8. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^5+4}}$

Use the Direct Comparison Test to determine the convergence or divergence of the series.

#10. $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

#11. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-1}$

#12. $\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$

Use the Limit Comparison Test to determine the convergence or divergence of the series.

#13. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

#14. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

#15. $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$

#16. Which test (nth-Term, Geometric, p-Series, Integral, Direct Comparison, or Limit Comparison) would you use to determine the convergence of the series? (Don't actually perform the test, just state which test is best…each test must be used at least once.)

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$$

$$\sum_{n=0}^{\infty} 5\left(-\frac{4}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \frac{1}{5^n+1}$$

$$\sum_{n=1}^{\infty} 2\frac{1}{n^3-8}$$

$$\sum_{n=1}^{\infty} \frac{2n}{3n-2}$$

$$\sum_{n=1}^{\infty} \frac{n}{\left(n^2+1\right)^2}$$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

7.5 – Required Practice

Determine the convergence or divergence of the series.

#1. Example: $\sum_{n=1}^{\infty}(-1)^n \frac{n}{2^{n-1}}$

#2. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^2}{n^2+4}$

Determine the convergence or divergence of the series.

#3. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\ln(n)}{n}$

#4. $-\frac{1}{3}+\frac{2}{4}-\frac{3}{5}+\frac{4}{6}-\frac{5}{7}+\ldots$

Determine the convergence or divergence of the series.

#5. $\sum_{n=1}^{\infty} \frac{4}{n(n+4)}$

#6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$

Determine the convergence or divergence of the series.

#7. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{3^n}$

#8. $\sum_{n=1}^{\infty}(-1)^n \frac{5n-1}{4n+1}$

#9. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\ln(n)}{n}$

Determine the convergence or divergence of the series.

#10. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$

#11. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n+1}{\ln(n+1)}$

#12. $\sum_{n=1}^{\infty}\sin\left(\frac{(2n+1)\pi}{2}\right)$

Determine the convergence or divergence of the series.

#13. $\sum_{n=0}^{\infty}(-1)^n \frac{1}{n!}$

#14. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{\sqrt{n}}{n+2}$

#15. $\sum_{n=1}^{\infty}\frac{6}{n(n+2)}$

7.6 – Required Practice

Determine whether the series coverges absolutely or conditionally, or diverges.

#1. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

#2. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+3}$

#3. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2n+3)}{n+10}$

Determine whether the series converges absolutely, conditionally, or diverges.

#4. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{2^n}$

Determine whether the series converges absolutely, conditionally, or diverges.

#5. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

#6. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{(n+1)^2}$

Determine whether the series converges absolutely, conditionally, or diverges.

#7. $\sum_{n=2}^{\infty}(-1)^n \frac{n}{n^3-5}$

7.7 – Required Practice

Determine whether the series converges absolutely, conditionally, or diverges.

#1. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

#2. $a_1 = 2, \quad a_{n+1} = \frac{5n+1}{4n+3} a_n$

$2, \ 2, \ 2.13333333, \ 2.357894....$

Determine whether the series converges absolutely, conditionally, or diverges.

#3. $\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{n+1}$

Determine whether the series converges absolutely, conditionally, or diverges.

#4. $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

#5. $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n$

Simplify the expression fully.

#6. $\dfrac{n5^{n+2}}{5^{n+1}}$

#7. $\dfrac{(n+1)!}{(n-2)!}$

Use the Ratio Test to determine the convergence or divergence of the series.

#8. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{5^n}$

#9. $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{3^n}$

#10. $\displaystyle\sum_{n=1}^{\infty}\frac{9^n}{n^5}$

#11. $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{n^5}$

Use the Ratio Test to determine the convergence or divergence of the series.

#12. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

#13. $\sum_{n=0}^{\infty} \frac{5^n}{2^n + 1}$

Use the Root Test to determine the convergence or divergence of the series.

#14. $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$

#15. $\sum_{n=2}^{\infty}\frac{(-1)^n}{(\ln(n))^n}$

#16. $\sum_{n=1}^{\infty}\left(2\sqrt[n]{n}+1\right)^n$

#17. $\sum_{n=0}^{\infty}e^{-3n}$

7.8 – Required Practice

#1. Example: Approximate the sum of the series by its first six terms: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n!}$

#2. Example: Determine the number of terms required to approximate the sum of the series with an error of less than 0.001: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^4}$

#3. How many terms of the series do we need to approximate the sum to an error of < 0.002? $\sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}$

Approximate the sum of the series using the first 6 terms.

#4. $\sum_{n=1}^{\infty} \frac{5(-1)^n}{n!}$

#5. $\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n^3}$

How many terms are required to approximate the series with an error of less than 0.001?

#6. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

Unit 7 Part 1 Test Review

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#1) $\sum_{n=1}^{\infty}\left(\frac{2n^3+1}{n^3-1}\right)^n$

#2) $\sum_{n=1}^{\infty}\frac{8^n}{5^n}$

#3) $\sum_{n=1}^{\infty}\frac{3^n}{4^n-1}$

#4) $\sum_{n=1}^{\infty}\frac{n}{\ln(n)}$

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#5) $\sum_{n=1}^{\infty} \frac{n!}{3^n}$

#6) $\sum_{n=1}^{\infty} \frac{1}{5^n + 1}$

#7) $\sum_{n=1}^{\infty} \frac{4n}{2n^2 + 1}$

#8) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#9) $\sum_{n=1}^{\infty} \frac{2n^3}{n^3+4}$

#10) $\sum_{n=1}^{\infty} (-1)^n \frac{5n-1}{4n+1}$

#11) $\sum_{n=0}^{\infty} 5\frac{2^n}{3^n}$

#12) $\sum_{n=1}^{\infty} \sin\left(\frac{(2n-1)\pi}{2}\right)$

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#13) $\sum_{n=1}^{\infty} \frac{1}{5^n}$

#14) $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

#15) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$

#16) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#17) $\sum_{n=1}^{\infty}(-2)^n$

#18) $\sum_{n=1}^{\infty}\frac{5n^4}{n^4+n^2+7}$

#19) $\sum_{n=1}^{\infty}\frac{9^n}{n^5}$

#20) $\sum_{n=1}^{\infty}\frac{5}{n^{0.4}}$

Determine whether the series diverges or converges (you must show work to justify using a valid and identify the test).

#21) $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^{n}$

#22) $\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{3^{n}}$

Determine whether the series converges absolutely, conditionally, or diverges:

#23) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}$

Determine whether the series converges absolutely, conditionally, or diverges:

#24) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$

#25) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{(n+1)^2}$

#26) Determine the minimum number of terms required to approximate the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n^3-1}$ with an error of less than 0.0005.