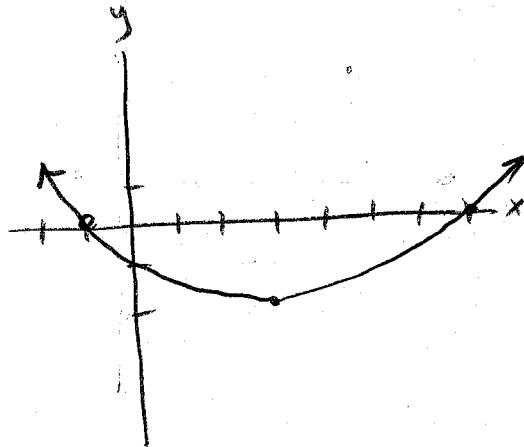
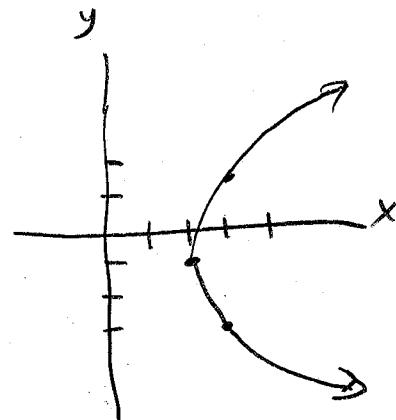


8.1 – Required Practice

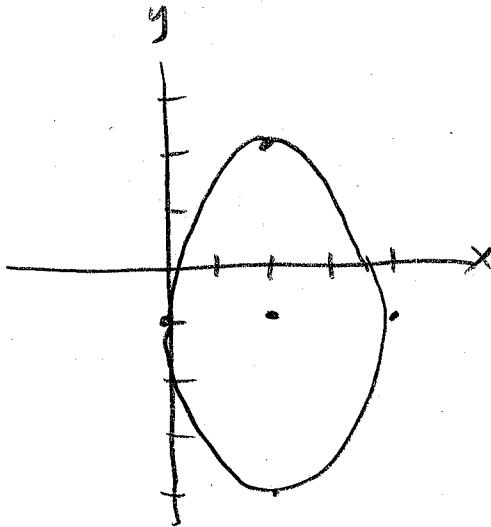
#1. $x^2 - 6x - 8y - 7 = 0$



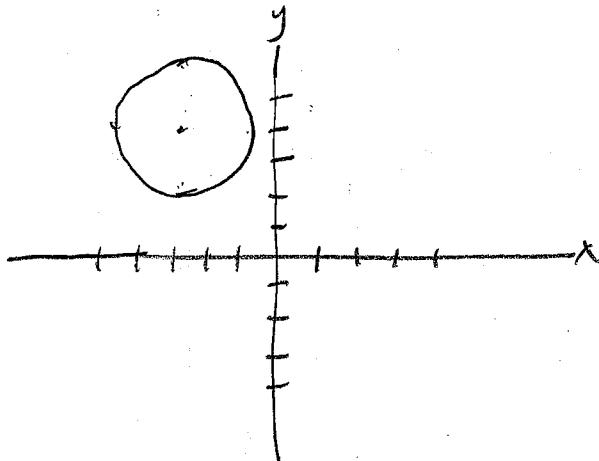
#2. $4x - y^2 - 2y - 9 = 0$



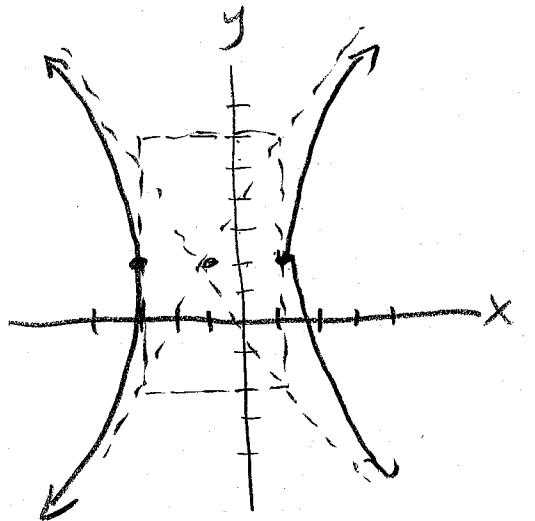
#3. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$



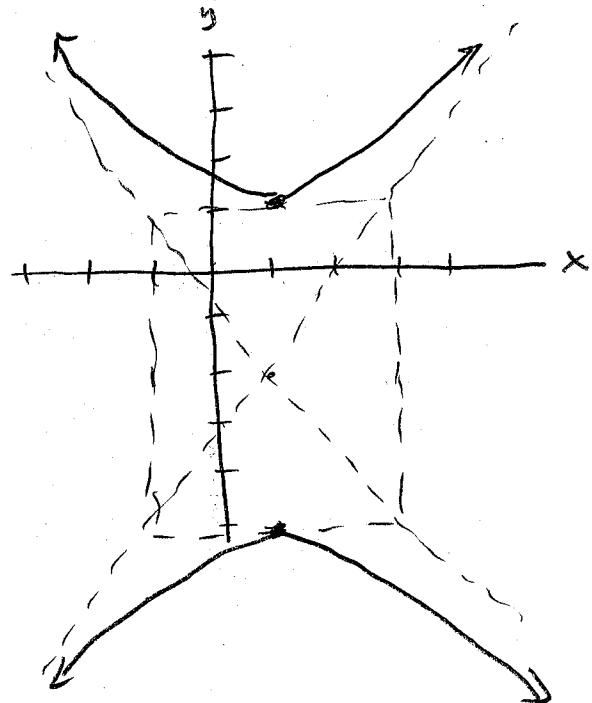
#4. $2x^2 + 2y^2 + 12x - 16y + 40 = 0$



$$\#5. 16x^2 - 4y^2 + 32x + 16y - 64 = 0$$



$$\#6. 9x^2 - 4y^2 - 18x - 16y + 29 = 0$$



Identify the type of each conic section.

$$\#7. x^2 + y^2 - 6x - 8y + 24 = 0$$

circle

$$\#8. 2y^2 + x - 12y + 20 = 0$$

parabola

$$\#9. x^2 + y^2 + 8y + 10 = 0$$

circle

$$\#10. 4x^2 - 9y^2 + 54y - 177 = 0$$

hyperbola

$$\#11. 49x^2 + y^2 - 294x + 392 = 0$$

ellipse

$$\#12. 9x^2 + 49y^2 + 98y - 392 = 0$$

ellipse

Identify the center and radius of each circle.

$$\#13. (x+8)^2 + (y-11)^2 = 25$$

$$\text{center} = (-8, 11)$$

$$r = 5$$

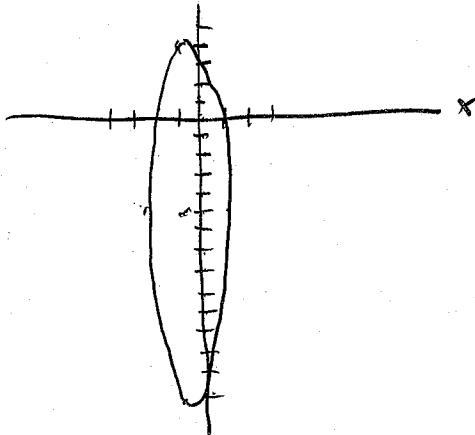
$$\#14. (x+8)^2 + (y+2)^2 = 67$$

$$\text{center} = (-8, -2)$$

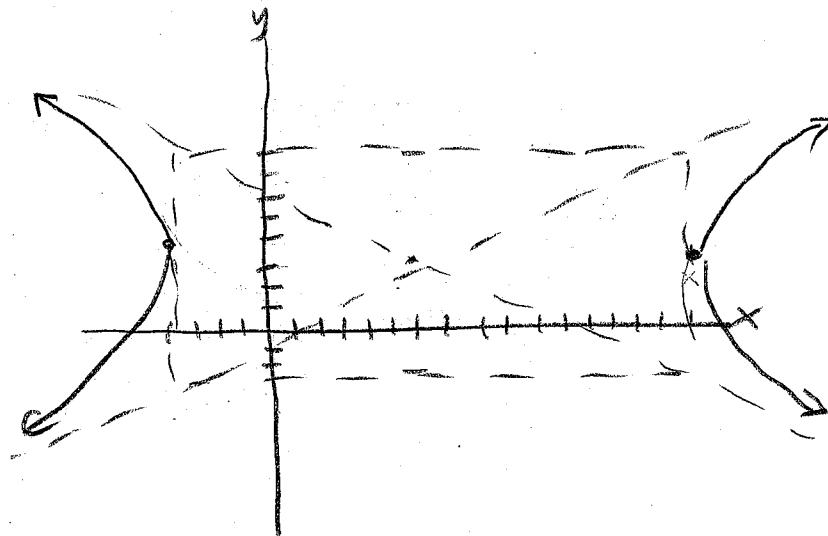
$$r = \sqrt{67}$$

Sketch the conic section already in standard form.

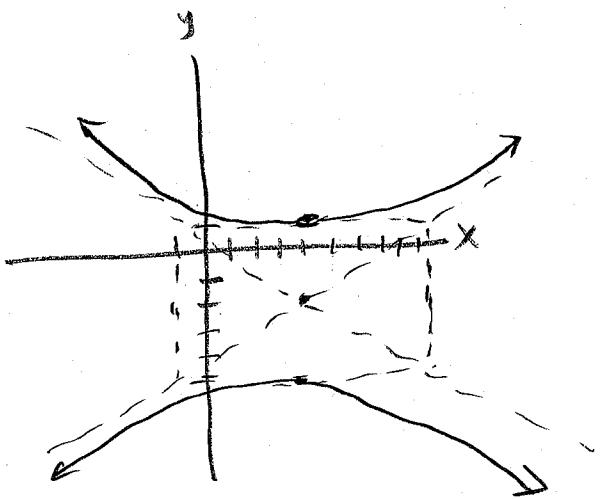
$$\#15. \frac{(x+1)^2}{4} + \frac{(y+5)^2}{81} = 1$$



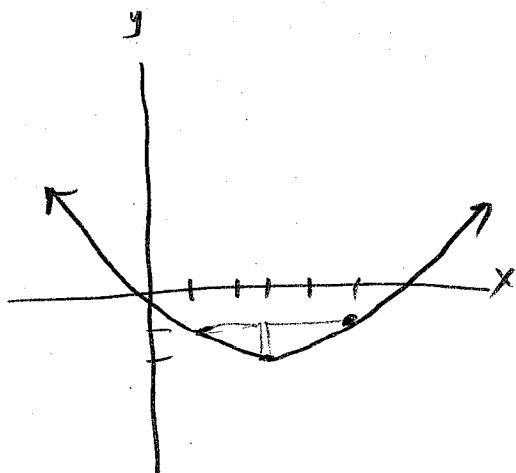
$$\#16. \frac{(x-6)^2}{100} - \frac{(y-3)^2}{25} = 1$$



$$\#17. \frac{(y+2)^2}{9} - \frac{(x-4)^2}{25} = 1$$



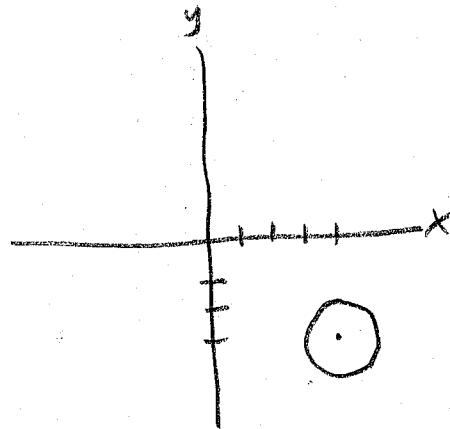
$$\#18. (x-3)^2 = 4(y+2)$$



Convert the equation to standard form and sketch.

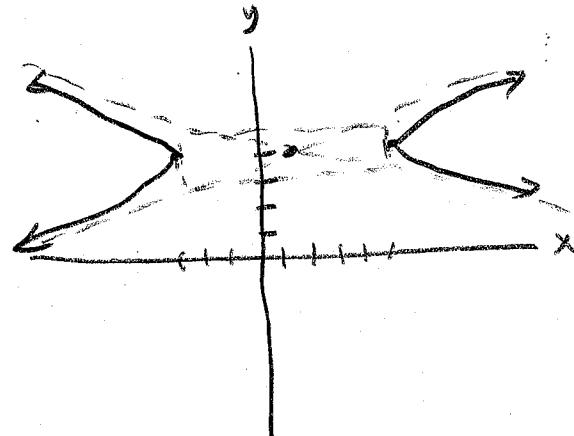
$$\#19. x^2 + y^2 - 8x + 6y + 24 = 0$$

$$(x-4)^2 + (y+3)^2 = 1$$



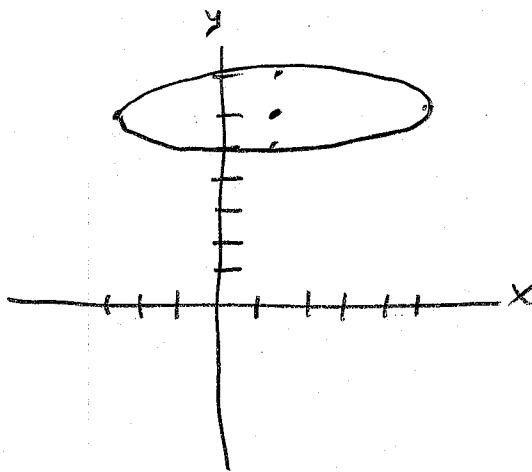
$$\#20. x^2 - 16y^2 - 2x + 128y - 271 = 0$$

$$\frac{(x-1)^2}{16} - \frac{(y-4)^2}{1} = 1$$



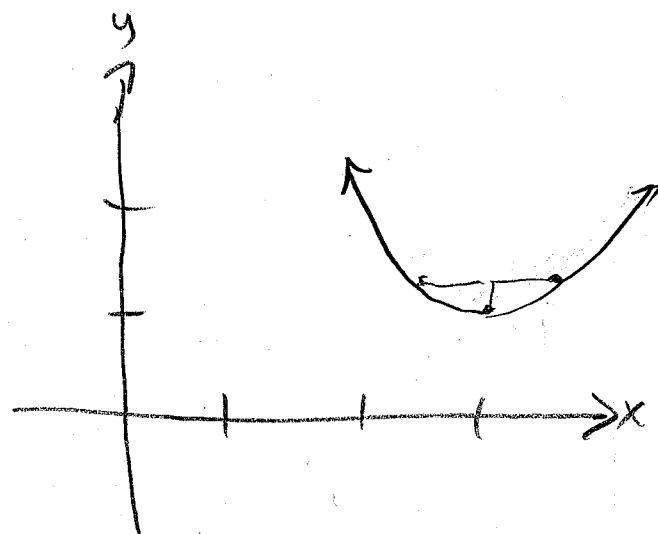
$$\#21. x^2 + 16y^2 - 2x - 192y + 561 = 0$$

$$\frac{(x-1)^2}{16} + (y-6)^2 = 1$$



$$\#22. -x^2 + 6x + y - 10 = 0$$

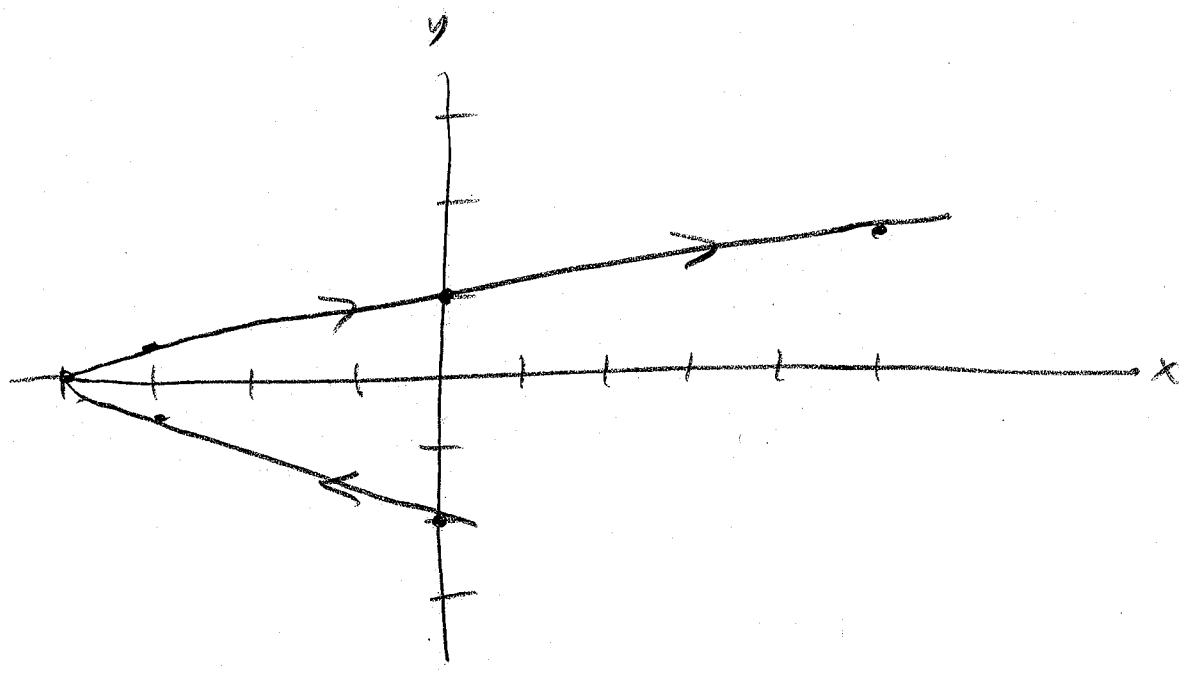
$$(x-3)^2 = 1(y-1)$$



8.2 – Required Practice

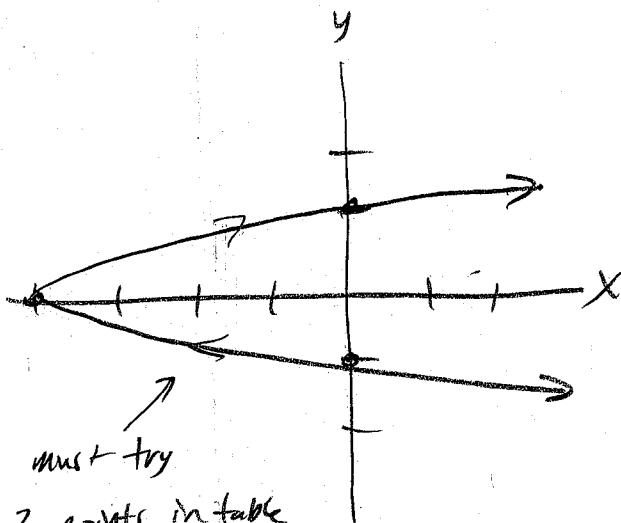
#1. Sketch the curve given by the parametric equations

$$\begin{cases} x = t^2 - 4 \\ y = \frac{1}{2}t \\ -2 \leq t \leq 3 \end{cases}$$



#2. $\begin{cases} x = t^2 - 4 \\ y = \frac{1}{2}t \end{cases}$

$\rightarrow 4y^2 = x + 4$

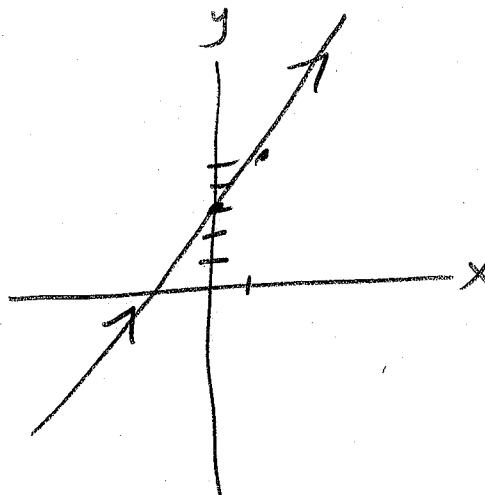


must try
2 points in table
to establish
direction and add arrows

#3.

$$\begin{cases} x = 2t \\ y = 4t + 3 \end{cases}$$

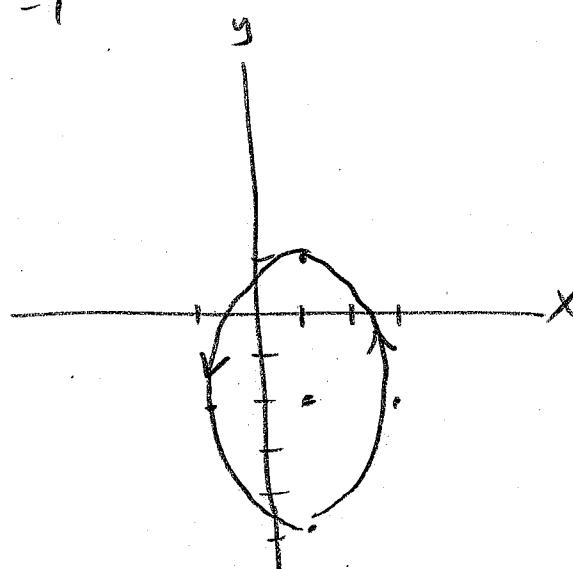
$$\rightarrow y = 2x + 3$$



#4.

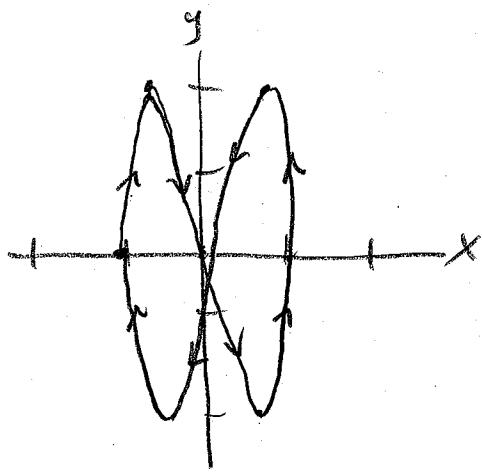
$$\begin{cases} x = 1 + 2\cos t \\ y = -2 + 3\sin t \end{cases}$$

$$\rightarrow \frac{(y+2)^2}{9} + \frac{(x-1)^2}{4} = 1$$



#5. Graph the plane curve and write the corresponding rectangular equation:

$$\begin{cases} x = \cos \theta \\ y = 2\sin(2\theta) \end{cases}$$



by table
or use $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\rightarrow y^2 = 16(1-x^2)x^2$$

or

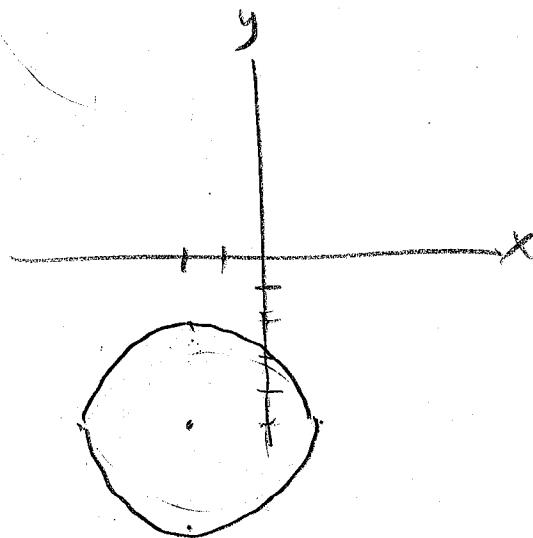
$$y = \pm 4x\sqrt{1-x^2}$$

#6. Graph the plane curve and write the corresponding rectangular equation:

$$\begin{cases} x = -2 + 3\cos\theta \\ y = -5 + 3\sin(\theta) \end{cases}$$

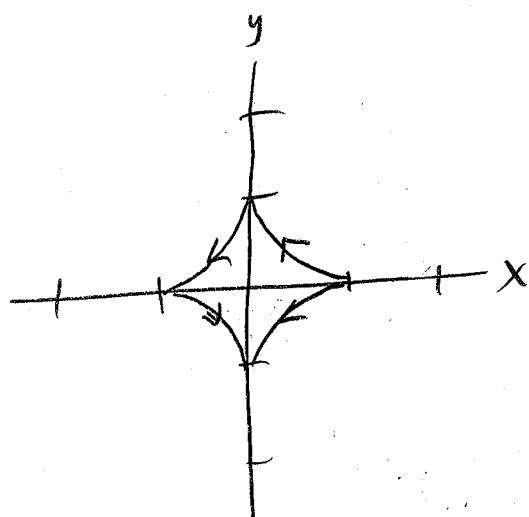
$$\rightarrow (y+5)^2 + (x+2)^2 = 9$$

(circle)



#7. Graph the plane curve and write the corresponding rectangular equation:

$$\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases}$$



by table

can convert to

$$y^{2/3} + x^{2/3} = 1$$

but doesn't help
w/sketch

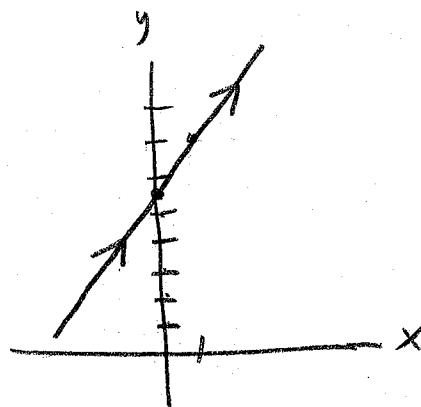
#8. Find two different sets of parametric equations for $y = x^3$

$x = t$	or	$x = \sqrt[3]{t}$
$y = t^3$		$y = t$

Sketch the curve of the parametric equation by either converting the equation to rectangular form, or using a table, then use your calculator to verify your sketch.

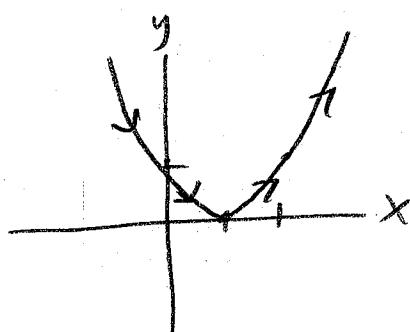
#9. $x = 2t - 3$, $y = 3t + 1$

$$\rightarrow y = \frac{3}{2}x + \frac{11}{2}$$



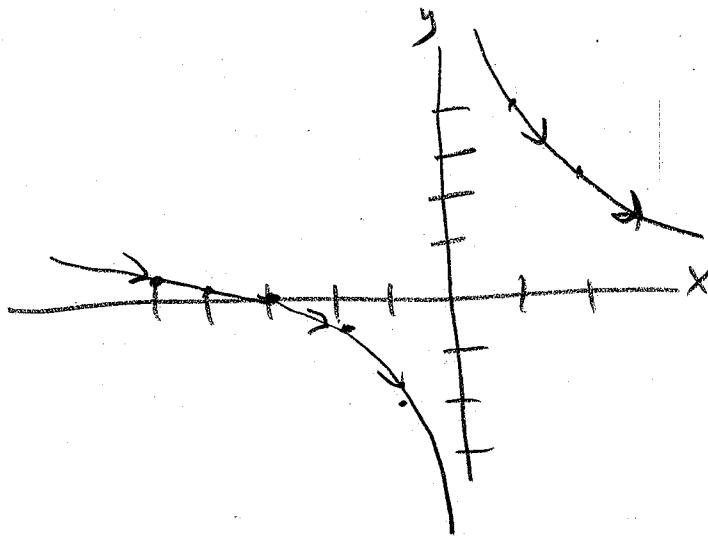
#10. $x = t + 1$, $y = t^2$

$$\rightarrow y = (x - 1)^2$$



#11. $x = t - 3$, $y = \frac{t}{t-3}$

used table

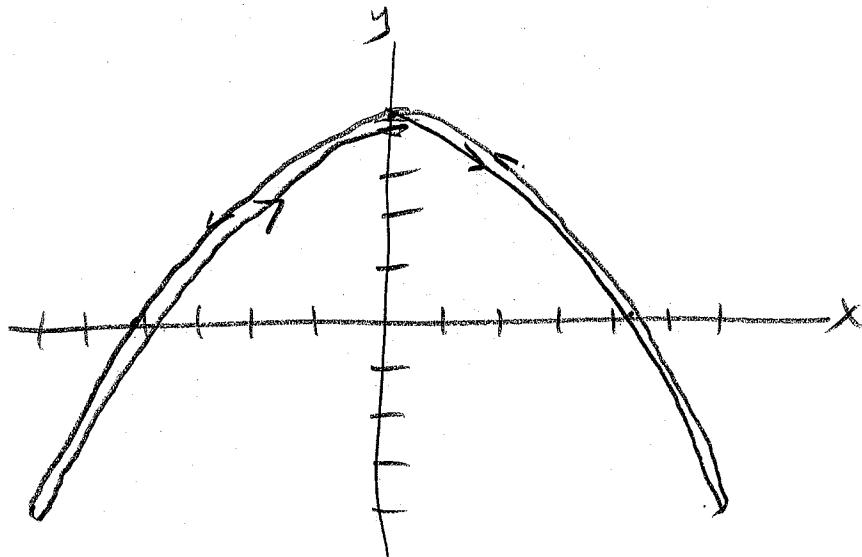


Sketch the curve of the parametric equation by either converting the equation to rectangular form, or using a table, then use your calculator to verify your sketch.

#12. $x = 6 \sin(\theta)$, $y = 4 \cos(2\theta)$

(Used table)

or calculator



#13. $x = 4 + 2 \cos(\theta)$, $y = -1 + \sin(\theta)$

$$\cos\theta = \frac{x-4}{2} \quad \sin\theta = y+1$$

$$\sin^2\theta + \cos^2\theta = 1$$

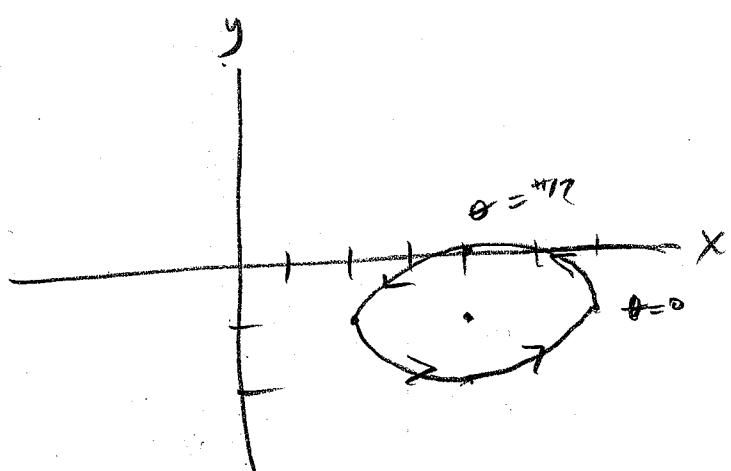
$$(y+1)^2 + \left(\frac{x-4}{2}\right)^2 = 1$$

$$\frac{(y+1)^2}{1} + \frac{(x-4)^2}{4} = 1$$

ellipse

center: $(4, -1)$

t	x	y
0	6	-1
$\frac{\pi}{2}$	4	0



8.3 day 1 – Required Practice

#1. A baseball is hit by a bat its trajectory is given by:

$$\begin{cases} x = t \\ y = 80 - \frac{77}{2500}(t-50)^2 \end{cases}$$

What is the angle of elevation of the path of the ball at $t = 0$, $t = 30$, and $t = 60$?

$$t=0: \theta = 72.013^\circ$$

$$t=30: \theta = 50.934^\circ$$

$$t=60: \theta = -31.633^\circ$$

Find $\frac{dy}{dx}$ for the given parametric equations.

#2. $x = t^2$, $y = 7 - 6t$

$$\boxed{\frac{dy}{dx} = \frac{-6}{2t} = \frac{-3}{t}}$$

#3. $x = \sin^2(\theta)$, $y = \cos^2(\theta)$

$$\boxed{\frac{dy}{dx} = -1}$$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and find the slope and concavity (if possible) at the given value of the parameter.

#4. $x = 4t$, $y = 3t - 2$ (at $t = 3$)

$$\boxed{\frac{dy}{dx} = \frac{3}{4}}$$
$$\frac{d^2y}{dx^2} = 0$$

#5. $x = 4\cos(\theta)$, $y = 4\sin(\theta)$ (at $t = \frac{\pi}{4}$)

$$\boxed{\frac{dy}{dx} = -1}$$
$$\frac{d^2y}{dx^2} = -\frac{1}{12}$$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and find the slope and concavity (if possible) at the given value of the parameter.

#6. $x = \sqrt{t}$, $y = \sqrt{t-1}$ (at $t=2$)

$$\frac{dy}{dx} = \frac{\sqrt{t}}{\sqrt{t-1}}, \quad \left. \frac{dy}{dx} \right|_{t=2} = \boxed{\sqrt{2}}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{(t-1)\sqrt{t-1}}, \quad \left. \frac{d^2y}{dx^2} \right|_{t=2} = \boxed{-1}$$

Find the equations of the tangent lines at the point where the curve crosses itself.

#7. $x = 2\sin(2t)$, $y = 3\sin(t)$

$y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$

Find all points (if any) of horizontal or vertical tangency to the curve.

#8. $x = 4 - t$, $y = t^2$

horizontal tangent at $(4, 0)$
vertical tangent nowhere

Find all points (if any) of horizontal or vertical tangency to the curve.

#9. $x = 3\cos(\theta)$, $y = 3\sin(\theta)$

horizontal tangents at $(0, 3)$ and $(0, -3)$

vertical tangents at $(3, 0)$ and $(-3, 0)$

Determine the open t-intervals on which the curve is concave up or concave down.

#10. $x = 2t + \ln(t)$, $y = 2t - \ln(t)$

Concave up: $(0, \infty)$

Concave down: Nowhere

Note: answer would have been: up: $(-\infty, -\frac{1}{2}) (0, \infty)$
down: $(-\frac{1}{2}, 0)$

except x and y are defined using $\ln(t)$
which means you can't use any t value < 0

8.3 day 2 – Required Practice

#1. Find the arc length of the curve on the given interval.

$$x = 6t^2, \quad y = 2t^3 \quad 1 \leq t \leq 4$$

$$\begin{aligned} \text{arc length} &= \int_1^4 \sqrt{(6t)^2 + (6t^2)^2} dt = 156.525 \text{ (calculator)} \\ &= 2(20)^{3/2} - 2(5)^{3/2} \text{ (by hand)} \end{aligned}$$

Find the arc length of the curve on the given interval.

#2. $x = 3t + 5, \quad y = 7 - 2t \quad (-1 \leq t \leq 3)$

$$\text{arc length} = \int_{-1}^3 \sqrt{(3)^2 + (-2)^2} dt = 4\sqrt{13} \quad (\text{eval by hand})$$

#3. $x = e^{-t} \cos(t), \quad y = e^{-t} \sin(t) \quad \left(0 \leq t \leq \frac{\pi}{2}\right)$

$$\begin{aligned} \text{arc length} &= \int_0^{\pi/2} \sqrt{-e^{-t} [smt + cost]^2 + [e^{-t} (cost - sint)]^2} dt \\ &= \int_0^{\pi/2} \sqrt{2} e^{-t} dt = \boxed{-\sqrt{2} [e^{-\pi/2} - e^0]} \end{aligned}$$

#4. An AP Exam Free-Response Question:

At time $t \geq 0$, the position of the particle moving along a curve in the xy-plane is $(x(t), y(t))$, where

$$\frac{dx}{dt} = 2t - 5\cos(t) \text{ and } \frac{dy}{dt} = -\sin(t). \text{ At time } t = 4, \text{ the particle is at the point } (-1, 3).$$

- Write an equation for the tangent line to the path of the particle at time $t = 4$.
- Find the time t when the tangent line to the path of the particle is vertical. Is the direction of the motion of the particle up or down at that moment? Explain your reasoning.
- Find the y-coordinate of the position of the particle at time $t = 0$.
- Find the total distance traveled by the particle on the interval $0 \leq t \leq 4$.

(Use calculator methods throughout this problem)

(a)
$$(y-3) = 0.067(x+1)$$

(b) vertical tangent at $t=1.111$ because $\frac{dx}{dt} > 0$

Since $\frac{dy}{dt} < 0$ at $t=1.11$, the particle is moving down

(c)
$$y(0) = 4.654$$

(d) Total dist = arclength = 26.656

Unit 8 Part 1 Test Review

Identify the conic, put the equation in standard form, and sketch:

#1. $x^2 - 6x - 8y - 7 = 0$ (parabola)

$$(x^2 - 6x + \underline{\underline{9}}) = 8y + 7 + \underline{\underline{9}}$$

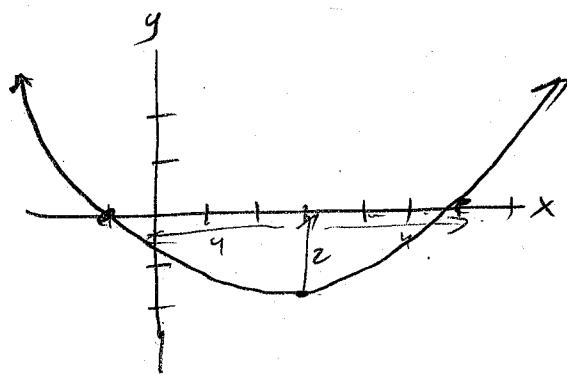
$$(x-3)^2 = 8y + 16$$

$$(x-3)^2 = 8(y+2)$$

vertex: $(3, -2)$

$$4p = 8 \quad (y = x^2)$$

$$p = 2 \quad \checkmark$$



#2. $y^2 + 16x + 2y - 63 = 0$ (parabola)

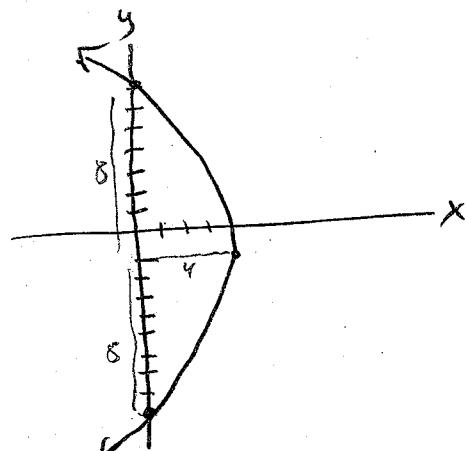
$$(y^2 + 2y + \underline{\underline{1}}) = -16x + 63 + \underline{\underline{1}}$$

$$(y+1)^2 = -16x + 64 = -16(x-4)$$

vertex: $(4, -1)$

$$4p = -16 \quad x = -y^2$$

$$p = -4 \quad \checkmark$$



#3. $3x^2 + 3y^2 + 12x + 18y + 12 = 0$ (circle)

$$(3x^2 + 12x + \underline{\underline{12}}) + (3y^2 + 18y + \underline{\underline{36}}) = -12$$

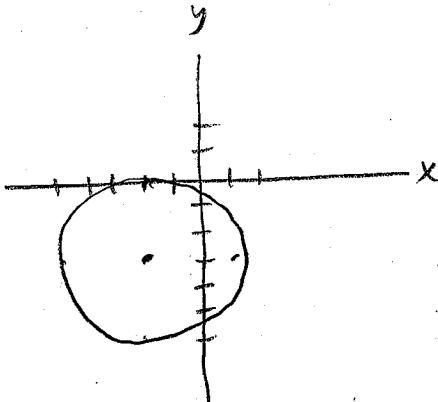
$$3(x^2 + 4x + \underline{\underline{4}}) + 3(y^2 + 6y + \underline{\underline{9}}) = -12 + \underline{\underline{12}} + \underline{\underline{27}}$$

$$3(x+2)^2 + 3(y+3)^2 = 27$$

$$(x+2)^2 + (y+3)^2 = 9$$

center: $(-2, -3)$

$$r = 3$$



#4. $2x^2 + 2y^2 - 16x + 4y + 24 = 0$ (circle)

$$(2x^2 - 16x + \underline{\underline{16}}) + (2y^2 + 4y + \underline{\underline{4}}) = -24$$

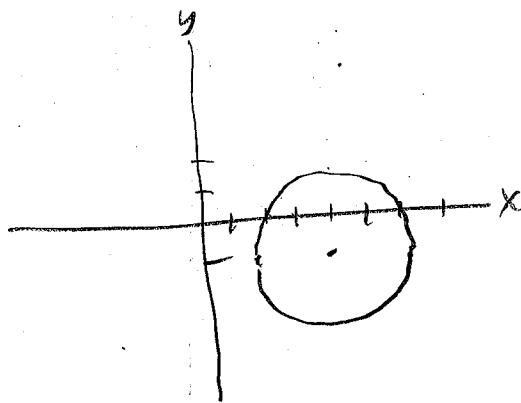
$$2(x^2 - 8x + \underline{\underline{16}}) + 2(y^2 + 2y + \underline{\underline{1}}) = -24 + \underline{\underline{32}} + \underline{\underline{2}}$$

$$2(x-4)^2 + 2(y+1)^2 = 10$$

$$(x-4)^2 + (y+1)^2 = 5$$

center: $(4, -1)$

$$r = \sqrt{5} (= 2.236)$$



Identify the conic, put the equation in standard form, and sketch:

#5. $9x^2 + 4y^2 + 18x - 16y - 11 = 0$ (ellipse)

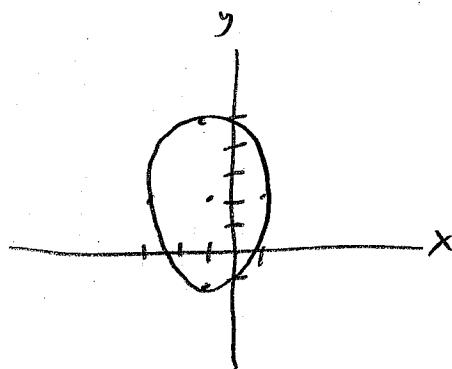
$$(9x^2 + 18x) + (4y^2 - 16y) = 11$$

$$9(x^2 + 2x + \underline{\underline{1}}) + 4(y^2 - 4y + \underline{\underline{4}}) = 11 + \underline{\underline{9}} + \underline{\underline{16}}$$

$$9(x+1)^2 + 4(y-2)^2 = 36$$

$$\frac{(x+1)^2}{9} + \frac{(y-2)^2}{4} = 1$$

center $(-1, 2)$



#6. $2x^2 + 50y^2 - 20x + 300y + 450 = 0$ (ellipse)

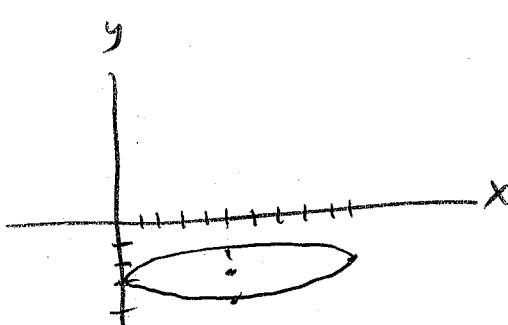
$$(2x^2 - 20x) + (50y^2 + 300y) = -450$$

$$2(x^2 - 10x + \underline{\underline{25}}) + 50(y^2 + 6y + \underline{\underline{9}}) = -450 + \underline{\underline{50}} + \underline{\underline{450}}$$

$$2(x-5)^2 + 50(y+3)^2 = 50$$

$$\frac{(x-5)^2}{25} + \frac{(y+3)^2}{1} = 1$$

center $(5, -3)$



#7. $4x^2 - 9y^2 + 16x + 54y - 101 = 0$ (hyperbola)

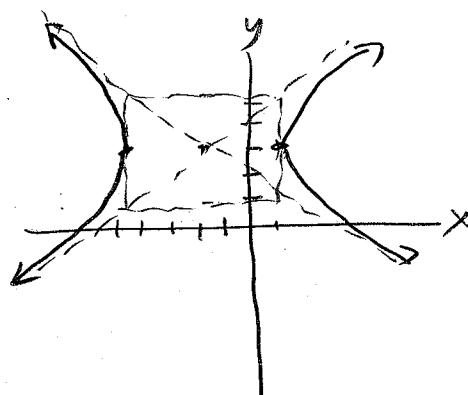
$$(4x^2 + 16x) + (-9y^2 + 54y) = 101$$

$$4(x^2 + 4x + \underline{\underline{4}}) - 9(y^2 - 6y + \underline{\underline{9}}) = 101 + \underline{\underline{16}} - \underline{\underline{81}}$$

$$4(x+2)^2 - 9(y-3)^2 = 32$$

$$\frac{(x+2)^2}{8} - \frac{(y-3)^2}{4} = 1$$

center $(-2, 3)$



#8. $9x^2 - 25y^2 - 54x - 50y + 281 = 0$ (hyperbola)

$$(9x^2 - 54x) + (-25y^2 - 50y) = -281$$

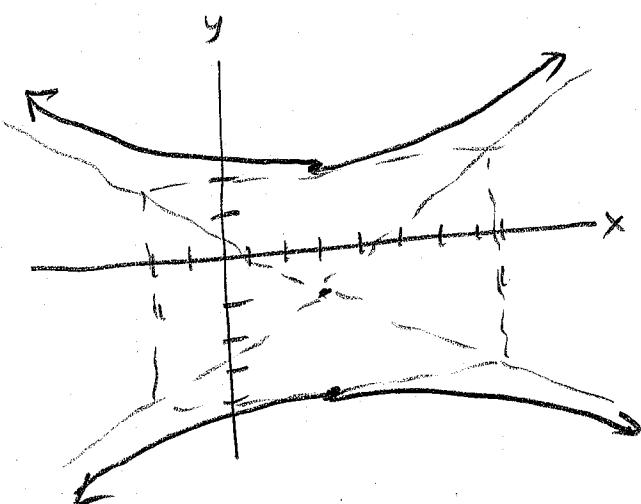
$$9(x^2 - 6x + \underline{\underline{36}}) - 25(y^2 + 2y + \underline{\underline{1}}) = -281 + \underline{\underline{324}} - \underline{\underline{25}}$$

$$9(x-3)^2 - 25(y+1)^2 = -225$$

$$25(y+1)^2 - 9(x-3)^2 = 225$$

$$\frac{(y+1)^2}{9} - \frac{(x-3)^2}{25} = 1$$

center $(3, -1)$



Convert the equation to rectangular form and sketch the curve (include direction arrows):

#9. $x = 2t, y = t^2 + 3$

$$t = \frac{1}{2}x, y = \left(\frac{1}{2}x\right)^2 + 3$$

$$y = \frac{1}{4}x^2 + 3$$

$$\frac{1}{4}x^2 = y - 3$$

$$x^2 = 4(y - 3)$$

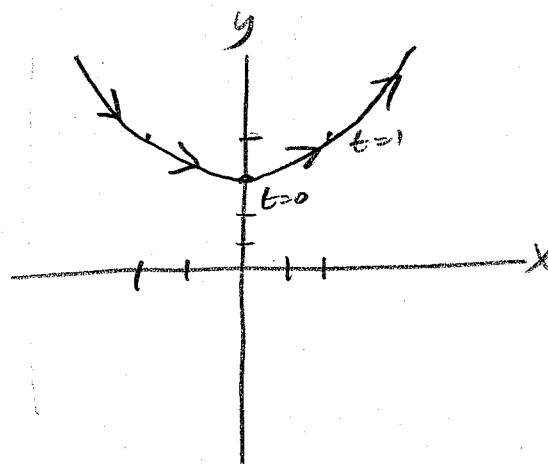
$$(x-0)^2 = 4(y-3)$$

vertex: $(0, 3)$

$$4p = 4 \quad p = 1$$

$$y = x^2$$

t	x	y
0	0	3
1	2	4



#10. $x = 3\cos t, y = 5\sin t$

$$\cos t = \frac{x}{3}, \sin t = \frac{y}{5}$$

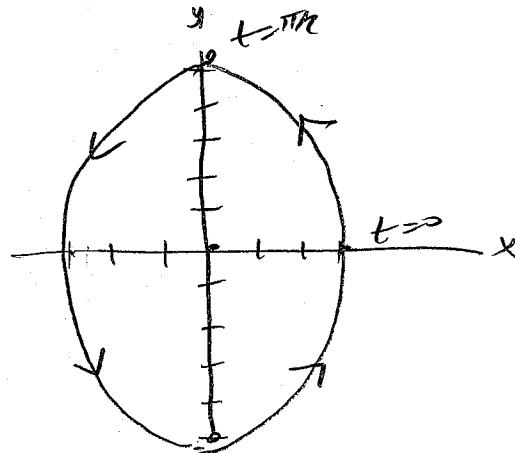
$$\cos^2 t + \sin^2 t = 1$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$

$$\frac{(x-0)^2}{9} + \frac{(y-0)^2}{25} = 1$$

center $(0, 0)$

t	x	y
0	3	0
$\pi/2$	0	5



#11. $x = 2 - 3\cos\theta, y = -3 + 3\sin\theta$

$$3\cos\theta = 2 - x \quad 3\sin\theta = y + 3$$

$$\cos\theta = \frac{2-x}{3} \quad \sin\theta = \frac{y+3}{3}$$

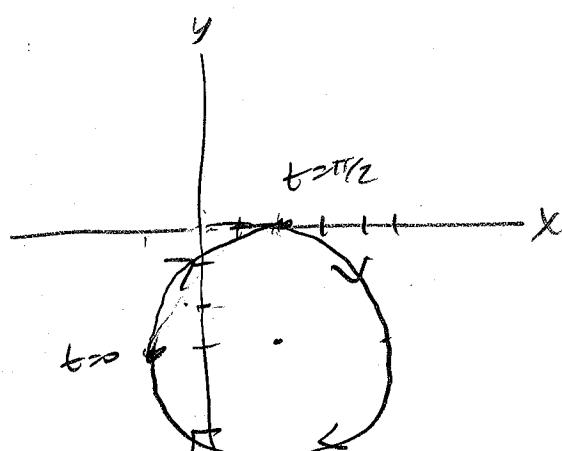
$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{2-x}{3}\right)^2 + \left(\frac{y+3}{3}\right)^2 = 1$$

$$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{9} = 1$$

center $(2, -3)$

t	x	y
0	-1	-3
$\pi/2$	2	0



Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve given by the parametric equations set, then find the slope and concavity at the given parameter value:

#12. $x = 4t^3 + t^2 - 2, \quad y = t^2 - 1 \quad \text{at } t = 2$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t}{12t^2 + 2t} = \frac{2t(1)}{2t(6t+1)} = \boxed{\frac{1}{6t+1}} = (6t+1)^{-1} \quad \text{slope} \Big|_{t=2} = \frac{1}{6(2)+1} = \boxed{\frac{1}{13}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{-(6t+1)^{-2}/6}{12t^2 + 2t} = \frac{-6}{(12t^2 + 2t)(6t+1)^2}$$

$$\text{concavity} \Big|_{t=2} = \frac{-6}{(12(2)^2 + 2(2))(6(2)+1)^2} = \frac{-6}{(52)(69)} = \boxed{-\frac{3}{4394}}$$

#13. $x = \ln(t), \quad y = \frac{1}{t^2} \quad \text{at } t = 1$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-2t^{-3}}{\frac{1}{t}} = -2t^{-3} = -2t^{-2} = \frac{-2}{t^2}$$

$$\text{slope} \Big|_{t=1} = \frac{-2}{(1)^2} = \boxed{-2}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) = \frac{4t^{-3}}{\left(\frac{1}{t}\right)} = 4t^{-3} = \frac{4t^{-3}}{t^1} = 4t^{-2} = \frac{4}{t^2}$$

$$\text{concavity} \Big|_{t=1} = \frac{4}{(1)^2} = \boxed{4}$$

Find the arc length of the curve on the given interval:

$$\#14. \quad x = 3\cos^2 t, \quad y = e^{2t} \quad 1 \leq t \leq 4$$

$$\begin{aligned}\frac{dx}{dt} &= 3(-2\cos t)(-\sin t) = 6\cos t \sin t \\ &= -6 \cos t \sin t\end{aligned}$$

$$\text{arc length} = \int_1^4 \sqrt{(-6 \cos t \sin t)^2 + (2e^{2t})^2} dt = 2973.632$$

(can't eval by hand, use math 9)

$$\#15. \quad x = \ln(t^2), \quad y = t^3 + 2 \quad 1 \leq t \leq 2$$

$$\frac{dx}{dt} = \frac{1}{t^2}(2t) = \frac{2t}{t^2} = \frac{2}{t} \quad \frac{dy}{dt} = 3t^2$$

$$\text{arc length} = \int_1^2 \sqrt{\left(\frac{2}{t}\right)^2 + (3t^2)^2} dt = 71.87$$

$$\begin{aligned}&\int_1^2 \sqrt{\frac{4}{t^2} + 9t^4} dt \\&\int_1^2 \sqrt{\frac{4}{t^2} + \frac{9t^6}{t^2}} dt \\&\int_1^2 \sqrt{\frac{4+9t^6}{t^2}} dt\end{aligned}$$

(too hard for by hand)

Find an equation of the tangent line to the curve at the given value of the parameter:

$$\#16. \quad x = 2\cos(t), \quad y = 3\sin(t) \quad \text{at } t = \frac{\pi}{6}$$

$$m = \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3\cos t}{-2\sin t} \Big|_{t=\frac{\pi}{6}} = \frac{3\cos\left(\frac{\pi}{6}\right)}{-2\sin\left(\frac{\pi}{6}\right)} = \frac{3\left(\frac{\sqrt{3}}{2}\right)}{-2\left(\frac{1}{2}\right)} = -\frac{3\sqrt{3}}{2}$$

$$x\left(\frac{\pi}{6}\right) = 2\cos\left(\frac{\pi}{6}\right) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$$

$$y\left(\frac{\pi}{6}\right) = 3\sin\left(\frac{\pi}{6}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$$

$$\boxed{(y - \frac{3}{2}) = -\frac{3\sqrt{3}}{2}(x - \sqrt{3})}$$

$$\#17. \quad x = t^3 + 4t^2, \quad y = 3t^{4/3} \quad \text{at } t = \frac{1}{8}$$

$$m = \frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4t^{1/3}}{3t^2 + 8t} \Big|_{t=\frac{1}{8}} = \frac{4\left(\frac{1}{8}\right)^{1/3}}{3\left(\frac{1}{8}\right)^2 + 8\left(\frac{1}{8}\right)} = \frac{4\left(\frac{1}{2}\right)}{\frac{3}{64} + 1} = \frac{2}{\left(\frac{67}{64}\right)} = \frac{128}{67} = \frac{128}{67}$$

$$x\left(\frac{1}{8}\right) = \left(\frac{1}{8}\right)^3 + 4\left(\frac{1}{8}\right)^2 = \frac{1}{512} + \frac{1}{16} = \frac{33}{512}$$

$$y\left(\frac{1}{8}\right) = 3\left(\frac{1}{8}\right)^{4/3} = 3\left(\frac{1}{2^4}\right) = 3\left(\frac{1}{16}\right) = \frac{3}{16}$$

$$\boxed{(y - \frac{3}{16}) = \frac{128}{64}(x - \frac{33}{512})}$$

(These are not on the test, but are good prep for calc 3 ☺)

Find a set of parametric equations which represents the given curve:

#18. A circle centered at (3, -4) with radius = 3, clockwise direction.

$$(x-3)^2 + (y+4)^2 = 9$$

$$\frac{(x-3)^2}{9} + \frac{(y+4)^2}{a^2} = 1$$

$$\left(\frac{x-3}{3}\right)^2 + \left(\frac{y+4}{3}\right)^2 = 1$$

$$\cos^2 t + \sin^2 t = 1$$

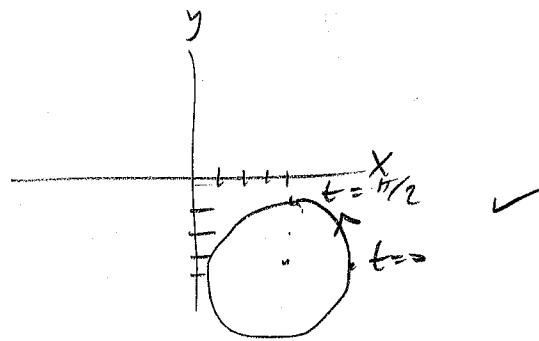
$$\cos t = \frac{x-3}{3} \quad \sin t = \frac{y+4}{3}$$

$$3\cos t = x-3 \quad 3\sin t = y+4$$

$x = 3 + 3\cos t$	$y = 3\sin t - 4$
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check direction

t	x	y
0	6	-4
$\frac{\pi}{2}$	3	-1



#19. An ellipse centered at the origin with major axis in y direction, $a = 3$, minor axis in x direction $b = 1$, and counter-clockwise direction.



$$\frac{(x-0)^2}{1} + \frac{(y-0)^2}{9} = 1$$

$$\frac{x^2}{1} + \frac{y^2}{9} = 1$$

$$\cos^2 t + \sin^2 t = 1$$

$$\sin t = x \quad \cos t = \frac{y}{3}$$

$$x = \sin t, \quad y = 3\cos t$$

check direction:

t	x	y
0	0	3
$\frac{\pi}{2}$	1	0

wrong direction,
Swap cos \leftrightarrow sin

$$(x)^2 + \left(\frac{y}{3}\right)^2 = 1$$

$$\cos^2 t + \sin^2 t = 1$$

$$\cos t = x \quad \sin t = \frac{y}{3}$$

$x = \cos t, \quad y = 3\sin t$

t	x	y
0	1	0
$\frac{\pi}{2}$	0	3

✓