AP Calculus BC – Unit 8 Required Practice **Name: ___________________________________**

8.1 – Required Practice

#1. $x^2 - 6x - 8y - 7 = 0$

#2. $4x - y^2 - 2y - 9 = 0$

#3. $9x^2 + 4y^2 - 36x + 8y + 4 = 0$

#4. $2x^2 + 2y^2 + 12x - 16y + 40 = 0$

#5. $16x^2 - 4y^2 + 32x + 16y - 64 = 0$

#6. $9x^2 - 4y^2 - 18x - 16y + 29 = 0$

Identify the type of each conic section.

#7. $x^2 + y^2 - 6x - 8y + 24 = 0$

#8. $2y^2 + x - 12y + 20 = 0$

#9. $x^2 + y^2 + 8y + 10 = 0$

#10. $4x^2 - 9y^2 + 54y - 177 = 0$

#11. $49x^2 + y^2 - 294x + 392 = 0$

#12. $9x^2 + 49y^2 + 98y - 392 = 0$

Identify the center and radius of each circle.

#13. $(x+8)^2+(y-11)^2=25$

#14. $(x+8)^2+(y+2)^2=67$

Sketch the conic section already in standard form.

#15. $\frac{(x+1)^2}{4}+\frac{(y+5)^2}{81}=1$

#16. $\frac{(x-6)^2}{100}-\frac{(y-3)^2}{25}=1$

#17. $\frac{(y+2)^2}{9}-\frac{(x-4)^2}{25}=1$

#18. $(x-3)^2=4(y+2)$

Convert the equation to standard form and sketch.

#19. $x^2 + y^2 - 8x + 6y + 24 = 0$

#20. $x^2 - 16y^2 - 2x + 128y - 271 = 0$

#21. $x^2 + 16y^2 - 2x - 192y + 561 = 0$

#22. $-x^2 + 6x + y - 10 = 0$

8.2 – Required Practice

#1. Sketch the curve given by the parametric equations

$$\begin{cases} x = t^2 - 4 \\ y = \dfrac{1}{2}t \end{cases}$$

$-2 \le t \le 3$

#2. $$\begin{cases} x = t^2 - 4 \\ y = \dfrac{1}{2}t \end{cases}$$

#3. $\begin{cases} x = 2t \\ y = 4t + 3 \end{cases}$

#4. $\begin{cases} x = 1 + 2\cos t \\ y = -2 + 3\sin t \end{cases}$

#5. Graph the plane curve and write the corresponding rectangular equation: $\begin{cases} x = \cos\theta \\ y = 2\sin(2\theta) \end{cases}$

#6. Graph the plane curve and write the corresponding rectangular equation: $\begin{cases} x = -2 + 3\cos\theta \\ y = -5 + 3\sin(\theta) \end{cases}$

#7. Graph the plane curve and write the corresponding rectangular equation: $\begin{cases} x = \cos^3 t \\ y = \sin^3 t \end{cases}$

#8. Find two different sets of parametric equations for $y = x^3$

Sketch the curve of the parametric equation by either converting the equation to rectangular form, or using a table, then use your calculator to verify your sketch.

#9. $x = 2t - 3, \quad y = 3t + 1$

#10. $x = t + 1, \quad y = t^2$

#11. $x = t - 3, \quad y = \dfrac{t}{t-3}$

Sketch the curve of the parametric equation by either converting the equation to rectangular form, or using a table, then use your calculator to verify your sketch.

#12. $x = 6\sin(\theta), \quad y = 4\cos(2\theta)$

#13. $x = 4 + 2\cos(\theta), \quad y = -1 + \sin(\theta)$

8.3 day 1 – Required Practice

#1. A baseball is hit by a bat its trajectory is given by: $\begin{cases} x = t \\ y = 80 - \dfrac{77}{2500}(t-50)^2 \end{cases}$

What is the angle of elevation of the path of the ball at t = 0, t = 30, and t = 60?

Find $\frac{dy}{dx}$ for the given parametric equations.

#2. $x = t^2, \quad y = 7 - 6t$

#3. $x = \sin^2(\theta), \quad y = \cos^2(\theta)$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and find the slope and concavity (if possible) at the given value of the parameter.

#4. $x = 4t, \quad y = 3t - 2 \quad (at \ \ t = 3)$

#5. $x = 4\cos(\theta), \quad y = 4\sin(\theta) \quad \left(at \ \ t = \frac{\pi}{4}\right)$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and find the slope and concavity (if possible) at the given value of the parameter.

#6. $x = \sqrt{t}, \quad y = \sqrt{t-1} \quad (at \ \ t = 2)$

Find the equations of the tangent lines at the point where the curve crosses itself.

#7. $x = 2\sin(2t), \quad y = 3\sin(t)$

Find all points (if any) of horizontal or vertical tangency to the curve.

#8. $x = 4 - t, \quad y = t^2$

Find all points (if any) of horizontal or vertical tangency to the curve.

#9. $x = 3\cos(\theta), \quad y = 3\sin(\theta)$

Determine the open t-intervals on which the curve is concave up or concave down.

#10. $x = 2t + \ln(t), \quad y = 2t - \ln(t)$

8.3 day 2 – Required Practice

#1. Find the arc length of the curve on the given interval.

$x = 6t^2,\ y = 2t^3 \qquad 1 \le t \le 4$

Find the arc length of the curve on the given interval.

#2. $x = 3t + 5, \quad y = 7 - 2t \quad (-1 \le t \le 3)$

#3. $x = e^{-t}\cos(t), \quad y = e^{-t}\sin(t) \quad \left(0 \le t \le \frac{\pi}{2}\right)$

#4. An AP Exam Free-Response Question:

At time $t \geq 0$, the position of the particle moving along a curve in the xy-plane is $\left(x(t), y(t)\right)$, where $\frac{dx}{dt} = 2t - 5\cos(t)$ and $\frac{dy}{dt} = -\sin(t)$. At time *t = 4*, the particle is at the point $(-1,3)$.

(a) Write an equation for the tangent line to the path of the particle at time *t = 4*.

(b) Find the time *t* when the tangent line to the path of the particle is vertical. Is the direction of the motion of the particle up or down at that moment? Explain your reasoning.

(c) Find the y-coordinate of the position of the particle at time *t = 0*.

(d) Find the total distance traveled by the particle on the interval $0 \leq t \leq 4$.

Unit 8 Part 1 Test Review

Identify the conic, put the equation in standard form, and sketch:

#1. $x^2 - 6x - 8y - 7 = 0$

#2. $y^2 + 16x + 2y - 63 = 0$

#3. $3x^2 + 3y^2 + 12x + 18y + 12 = 0$

#4. $2x^2 + 2y^2 - 16x + 4y + 24 = 0$

Identify the conic, put the equation in standard form, and sketch:

#5. $9x^2+4y^2+18x-16y-11=0$

#6. $2x^2+50y^2-20x+300y+450=0$

#7. $4x^2-9y^2+16x+54y-101=0$

#8. $9x^2-25y^2-54x-50y+281=0$

Convert the equation to rectangular form and sketch the curve (include direction arrows):

#9. $x = 2t,\ \ y = t^2 + 3$

#10. $x = 3\cos t, \quad y = 5\sin t$

#11. $x = 2 - 3\cos\theta, \quad y = -3 + 3\sin\theta$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve given by the parametric equations set, then find the slope and concavity at the given parameter value:

#12. $x=4t^3+t^2-2, \quad y=t^2-1$ at $t=2$

#13. $x=\ln(t), \quad y=\frac{1}{t^2}$ at $t=1$

Find the arc length of the curve on the given interval:

#14. $x = 3\cos^2 t, \quad y = e^{2t} \qquad 1 \le t \le 4$

#15. $x = \ln\left(t^2\right), \quad y = t^3 + 2 \qquad 1 \le t \le 2$

Find an equation of the tangent line to the curve at the given value of the parameter:

#16. $x = 2\cos(t), \quad y = 3\sin(t) \quad at\ t = \frac{\pi}{6}$

#17. $x = t^3 + 4t^2, \quad y = 3t^{4/3} \quad at\ t = \frac{1}{8}$

Find a set of parametric equations which represents the given curve:

#18. A circle centered at (3, -4) with radius = 3, clockwise direction.

#19. An ellipse centered at the origin with major axis in y direction, a = 3, minor axis in x direction b = 1, and counter-clockwise direction.