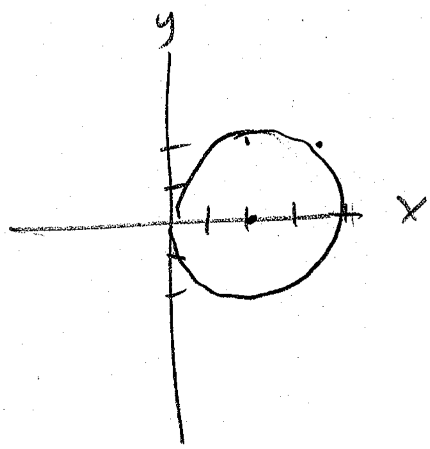
AP Calculus BC – Unit 8 Part 2 Required Practice

Name: answers only

8.4 – Required Practice

#1. Sketch $r = 4\cos\theta$

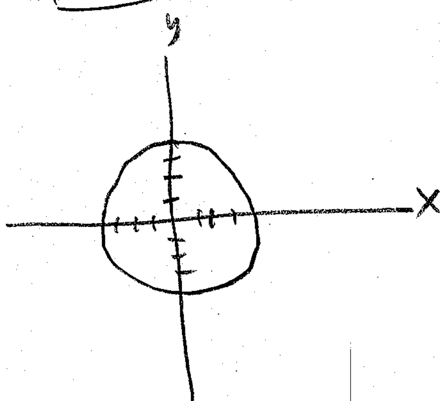


table,
Calculator in polar mode,
convert to rectangular
(try all 3)

Convert from rectangular to polar form and sketch:

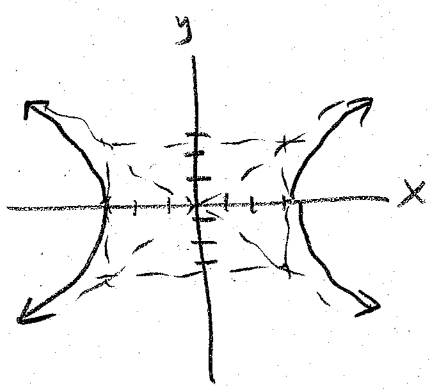
#2. $x^2 + y^2 = 16$

$\boxed{r = 4}$



#3. $x^2 - y^2 = 9$

$$\boxed{r = \sqrt{\frac{9}{\cos^2\theta - \sin^2\theta}} = \frac{3}{\sqrt{\cos^2\theta - \sin^2\theta}}}$$

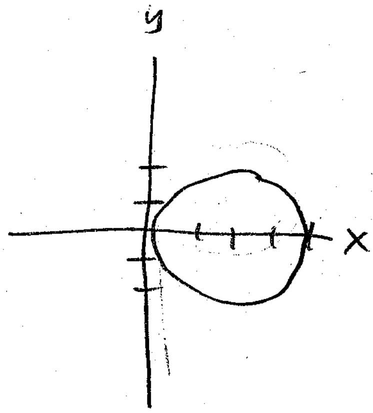


$\frac{x^2}{9} - \frac{y^2}{9} = 1$
(hyperbola)

Convert from rectangular to polar form and sketch:

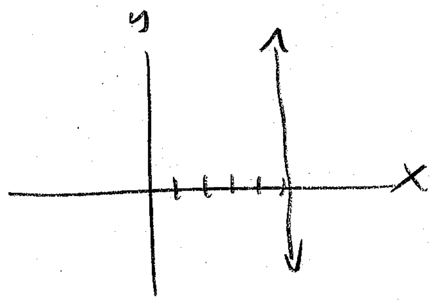
#4. $x^2 + y^2 - 4x = 0$

$\boxed{r = 4\cos\theta}$



#5. $x = 5$

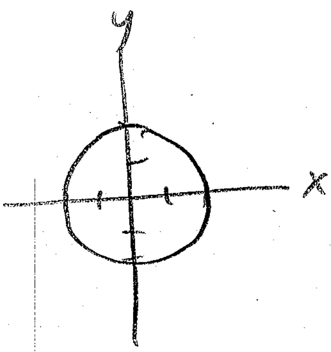
$\boxed{r = \frac{5}{\cos\theta} = 5\sec\theta}$



Convert from polar to rectangular form and sketch:

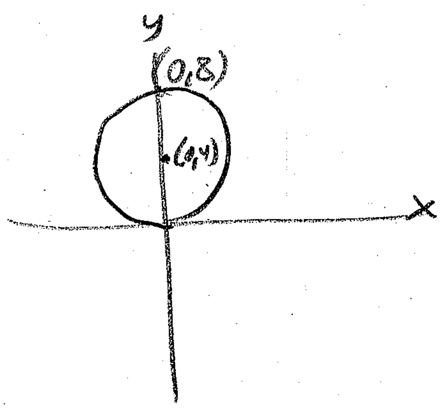
#6. $r = 2$

$\boxed{x^2 + y^2 = 4}$



#7. $r = 8\sin\theta$

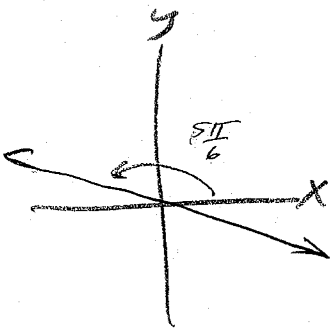
$\boxed{x^2 + y^2 = 8y}$



Convert from polar to rectangular form and sketch:

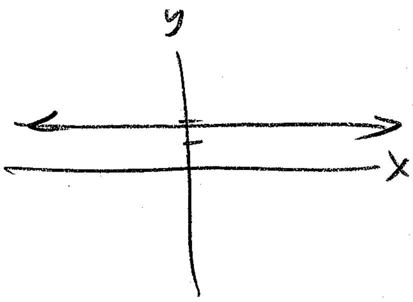
#8. $\theta = \dfrac{5\pi}{6}$

$\boxed{y = -\dfrac{1}{\sqrt{3}}x}$



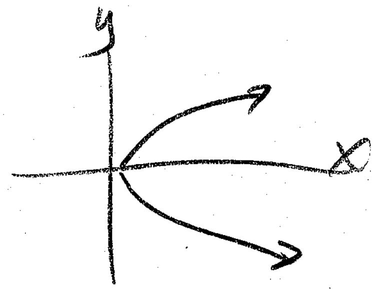
#9. $r = 2\csc\theta$

$\boxed{y = 2}$



#10. $r = \cot\theta\csc\theta$

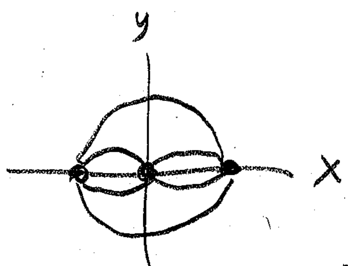
$\boxed{y^2 = x}$



Examples...Find the intersection points of the curves:

#11. $r = 3(1+\sin\theta)$

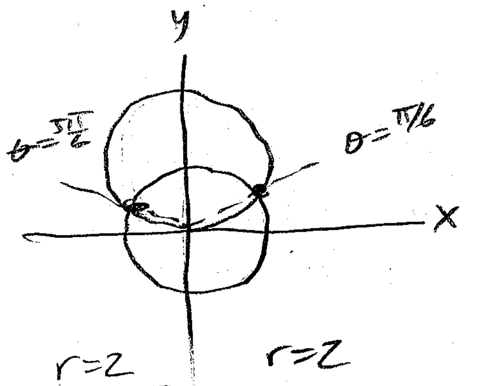
$r = 3(1-\sin\theta)$



$r=3$, $\theta=\pi$	$(r=0)$	$r=3$, $\theta=0$
$\boxed{(-3,0)}$	$\boxed{(0,0)}$	$\boxed{(3,0)}$

#12. $r = 4\sin\theta$

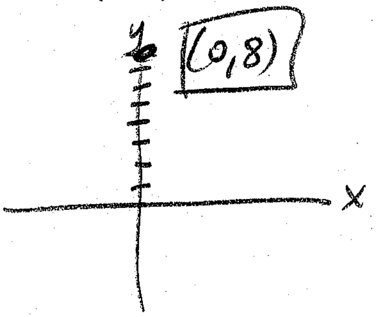
$r = 2$



$r=2$, $\theta=\frac{5\pi}{6}$	$r=2$, $\theta=\frac{\pi}{6}$
$\boxed{(-\sqrt{3},1)}$	$\boxed{(3,1)}$

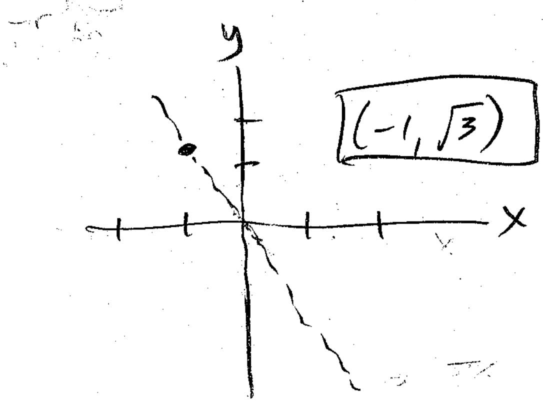
Plot the (r,θ) polar coordinate and find the corresponding rectangular (x,y) coordinate.

#13. $\left(8, \frac{\pi}{2}\right)$



$\boxed{(0,8)}$

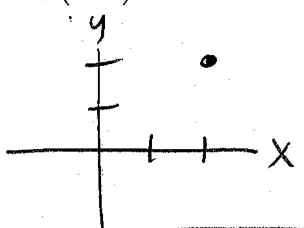
#14. $\left(-2, \frac{5\pi}{3}\right)$



$\boxed{(-1, \sqrt{3})}$

The rectangular (x,y) coordinate is given. Plot the coordinate, the find two sets of polar coordinates for the same location with $0 \le \theta < 2\pi$

#15. $(2,2)$

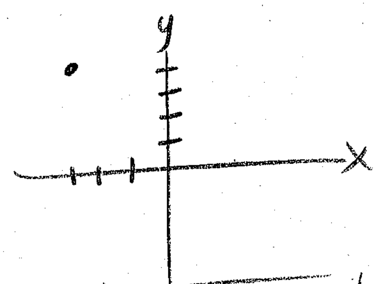


r θ

$(\sqrt{8}, \frac{\pi}{4})$

$(\sqrt{8}, \frac{-7\pi}{4})$

#16. $(-3,4)$



r θ

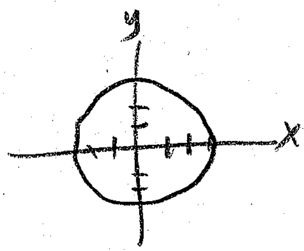
$(5, 2.214)$

$(5, -4.069)$

Convert the rectangular equation to polar form and sketch its graph.

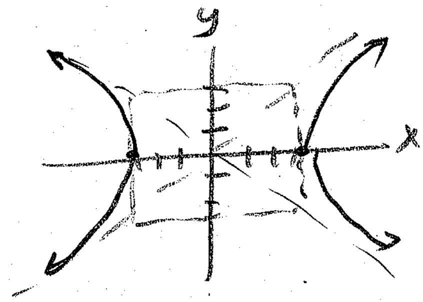
#17. $x^2+y^2=9$

$\boxed{r=3}$



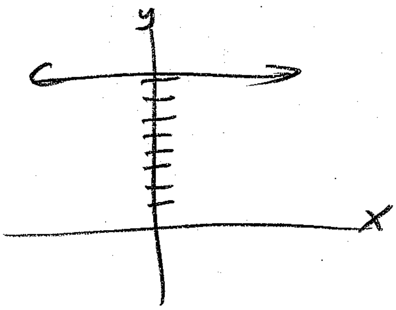
#18. $x^2-y^2=9$

$$\boxed{r=\frac{3}{\sqrt{\cos^2\theta-\sin^2\theta}}}$$



#19. $y=8$

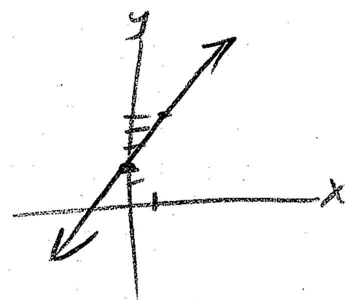
$$\boxed{r=\frac{8}{\sin\theta}=8\csc\theta}$$



#20. $3x-y+2=0$

$y=3x+2$

$$\boxed{r=\frac{-2}{3\cos\theta-\sin\theta}}$$

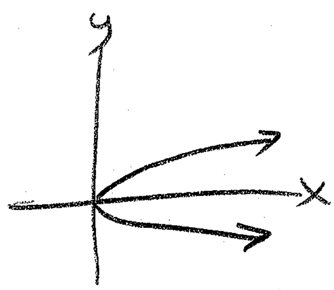


#21. $y^2=9x$

$\boxed{r=0}$ (origin)

or

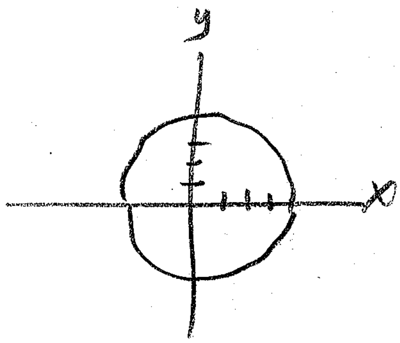
$$\boxed{r=\frac{9\cos\theta}{\sin^2\theta}}$$



Convert the polar equation to rectangular form and sketch its graph.

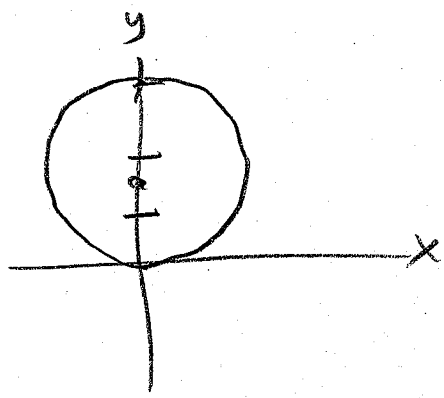
#22. $r=4$

$$\boxed{x^2+y^2=16}$$



#23. $r=3\sin(\theta)$

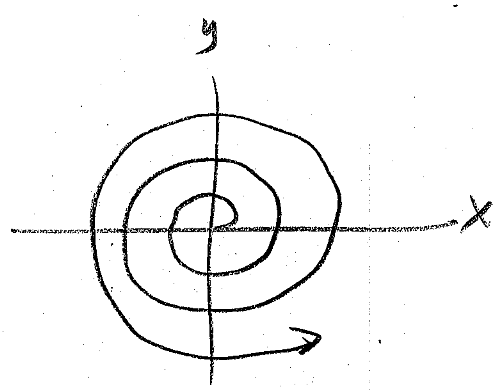
$$\boxed{\begin{gathered} x^2+y^2=3y \\ \text{or} \\ x^2+\left(y-\tfrac{3}{2}\right)^2=\tfrac{9}{4} \end{gathered}}$$



#24. $r=\theta$

$$\boxed{\tan\left(\sqrt{x^2+y^2}\right)=\frac{y}{x}}$$

(wow!)



Find the points of intersection of the graphs of the equations

#25. $r=1+\cos(\theta)$
$r=1-\cos(\theta)$

$\boxed{(0,1)\ (0,-1)\ (0,0)}$

#26. $r=4-5\sin(\theta)$
$r=3\sin(\theta)$

$\boxed{\left(\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)\left(-\frac{3\sqrt{3}}{4}, \frac{3}{4}\right)(0,0)}$

8.5 – Required Practice

#1. Find the points of horizontal and vertical tangency to the polar curve $r = 4\sin\theta$ and find the equation of the tangent line at $\theta = \frac{\pi}{3}$

vertical tangents at $(x,y) = (2,2)$ and $(-2,2)$

horizontal tangents at $(0,0)$ and $(4,0)$

tangent line: $(y-3) = (-\sqrt{3})(x-\sqrt{3})$

#2. Find the arc length of the top half of the cardioid $r = 2 - 2\cos\theta$

$$\boxed{\text{arc length} = \int_0^{\pi} \sqrt{(2-2\cos\theta)^2 + (2\sin\theta)^2}\, d\theta = 8}$$

#3. Find the arc length of the curve $r = 2\sin(3\theta)$

$$\boxed{\text{arc length} = \int_0^{\pi} \sqrt{(2\sin(3\theta))^2 + (6\cos(3\theta))^2}\, d\theta = 13.365}$$

Find the points of vertical and horizontal tangency (if any) to the polar curve.

#4. $r = 1 - \sin(\theta)$

$x \quad y$

vertical tangents at $\left(-\frac{3\sqrt{3}}{4}, -\frac{3}{4}\right)$ and $\left(\frac{3\sqrt{3}}{4}, -\frac{3}{4}\right)$

horizontal tangents at $\left(\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$ $\left(-\frac{\sqrt{3}}{4}, \frac{1}{4}\right)$ $(0, -2)$

Find the arc length.

#5. $r = 2\theta, \quad 0 \le \theta \le 2\pi$

$$\boxed{\text{arclength} = \int_0^{2\pi} \sqrt{(2\theta)^2 + (2)^2}\, d\theta = 42.513}$$

#6. One petal of $r = 4\cos(3\theta)$

$$\boxed{\text{arclength} = \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \sqrt{(4\cos(3\theta))^2 + (-12\sin(3\theta))^2}\, d\theta = 8.910}$$

limits could also be $\frac{3\pi}{6} \to \frac{5\pi}{6}$

$-\frac{\pi}{6} \to \frac{\pi}{6}$

$\frac{5\pi}{6} \to \frac{7\pi}{6}$

$\vdots$

8.6 – Required Practice

#1. Find the area in the interior of $r = 3\cos\theta$

$$\boxed{A = 2\left(\frac{1}{2}\right)\int_{0}^{\pi/2} (3\cos\theta)^2\,d\theta = \frac{9\pi}{4}}$$

#2. Find the area in the inner loop of $r = 1 + 2\sin\theta$

$$\boxed{A = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} (1+2\sin\theta)^2\,d\theta = 0.5435}$$

#3. Find the area inside $r = 3\sin\theta$ and outside $r = 1 + \sin\theta$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (3\sin\theta)^2 d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1+\sin\theta)^2 d\theta$$

$$= \pi$$

#4. Find the common interior area of $r = 2\cos\theta$ and $r = 2\sin\theta$

$$A = \frac{1}{2}\int_{0}^{\pi/4} (2\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} (2\cos\theta)^2 d\theta$$

$$= \frac{\pi}{2} - 1$$

Write (and evaluate with calculator) an integral that represents the indicated area.

#5. *interior of* $r=4\sin(\theta)$

$$\boxed{A=\frac{1}{2}\int_0^{\pi}(4\sin\theta)^2\,d\theta=12.566}$$

#6. *interior of* $r=3\cos(\theta)$

$$\boxed{A=\frac{1}{2}\int_{\frac{\pi}{2}}^{3\pi/2}(3\cos\theta)^2\,d\theta=7.069}$$

#7. *one petal of* $r=\sin(2\theta)$

$$\boxed{A=\frac{1}{2}\int_0^{\pi/2}(\sin(2\theta))^2\,d\theta=0.393}$$

#8. *inner loop of* $r=1+2\cos(\theta)$

$$\boxed{A=\frac{1}{2}\int_{2\pi/3}^{4\pi/3}(1+2\cos\theta)^2\,d\theta=0.544}$$

Write (and evaluate with calculator) an integral that represents the indicated area.

#9. *inside* $r=2\cos(\theta)$ *and outside* $r=1$

$$\boxed{A=\frac{1}{2}\int_{-\pi/3}^{\pi/3}(2\cos\theta)^2\,d\theta-\frac{1}{2}\int_{-\pi/3}^{\pi/3}(1)^2\,d\theta=1.913}$$

#10. *common interior of* $r=4\sin(\theta)$ *and* $r=2$

$$\boxed{A=\frac{1}{2}\int_{0}^{\pi/6}(4\sin\theta)^2\,d\theta+\frac{1}{2}\int_{\frac{\pi}{6}}^{5\pi/6}(2)^2\,d\theta+\frac{1}{2}\int_{\frac{5\pi}{6}}^{\pi}(4\sin\theta)^2\,d\theta=4.913}$$

Unit 8 Part 2 Test Review

Convert the equation to polar form and sketch the curve:

#1. $9x^2 + 9y^2 = 18y$

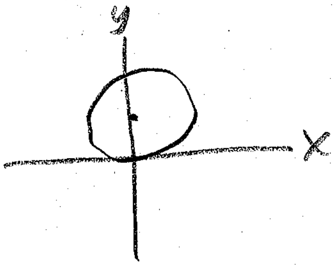
$9x^2 + 9y^2 - 18y = 0$

$9(x-0)^2 + 9(y^2 - 2y + 1) = 0 + 9$

$9(x-0)^2 + 9(y-1)^2 = 9$

$(x-0)^2 + (y-1)^2 = 1$

circle, center $= (0,1)$, $r = 1$



$9(x^2+y^2) = 18y$

$9r^2 = 18r\sin\theta$

$9r^2 - 18r\sin\theta = 0$

$9r(r - 2\sin\theta) = 0$

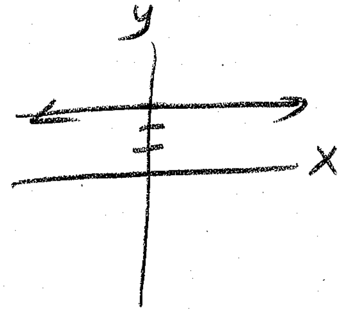
$\boxed{r=0}$ or

$\boxed{r = 2\sin\theta}$

#2. $y = 3$

$r\sin\theta = 3$

$$\boxed{\begin{array}{l} r = \frac{3}{\sin\theta} \\ r = 3\csc\theta \end{array}}$$



#3. $y = x^2$

$r\sin\theta = (r\cos\theta)^2$

$r\sin\theta = r^2\cos^2\theta$

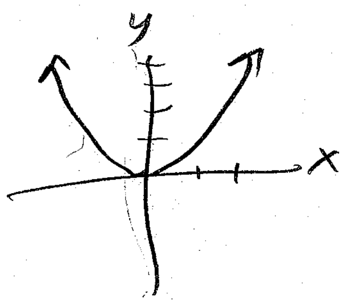
$r\sin\theta - r^2\cos^2\theta = 0$

$r(\sin\theta - r\cos^2\theta) = 0$

$\boxed{r=0}$ (origin) or $\sin\theta - r\cos^2\theta = 0$

$r\cos^2\theta = \sin\theta$

$$\boxed{r = \frac{\sin\theta}{\cos^2\theta}}$$

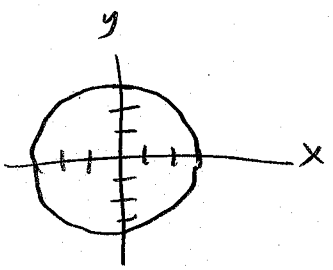


Convert the equation to rectangular form and sketch the curve:

#4. $r = 3$

$r^2 = 9$

$\boxed{x^2+y^2=9}$



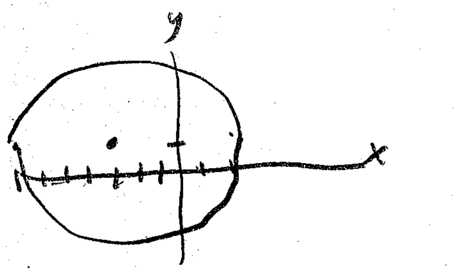
#5. $r + 6\cos\theta - 2\sin\theta = \frac{6}{r}$

$r^2 + 6r\cos\theta - 2r\sin\theta = 6$

$\boxed{x^2+y^2+6x-2y=6}$

$(x^2+6x+9) + (y^2-2y+1) = 6+9+1$

$\boxed{(x+3)^2+(y-1)^2=16}$ circle, center $(-3,1)$ $r=4$



#6. $r = 8\sin\theta$

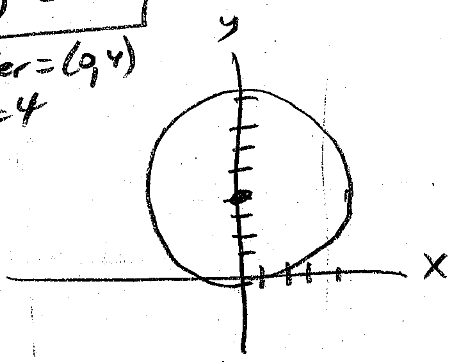
$r^2 = 8r\sin\theta$

$\boxed{x^2+y^2=8y}$

$x^2+(y^2-8y+16) = 0+16$

$\boxed{x^2+(y-4)^2=16}$

circle, center $=(0,4)$ $r=4$



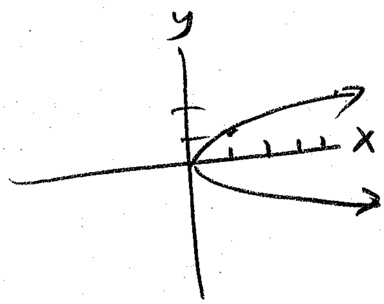
#7. $r = \cot\theta\csc\theta$

$r = \cot\theta \frac{1}{\sin\theta}$

$r\sin\theta = \cot\theta$

$y = \frac{x}{y}$

$\boxed{y^2 = x}$

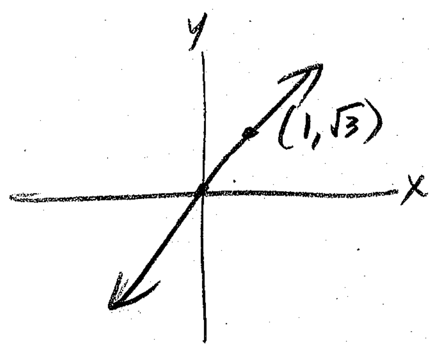


#8. $\theta = \frac{\pi}{3}$

$\tan(\theta) = \tan(\frac{\pi}{3})$

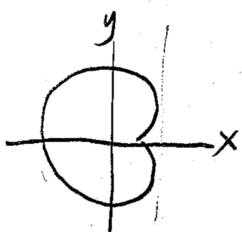
$\frac{y}{x} = \frac{\sin(\pi/3)}{\cos(\pi/3)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

$\boxed{y = \sqrt{3}x}$



Graph the polar equation curve and find an interval for which the graph is traced only once:

#9. $r = 4 - 3\cos\theta$

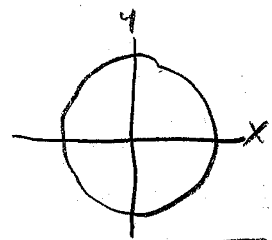


when $y=0$
$y = r\sin\theta$
$(4-3\cos\theta)\sin\theta = 0$
$\sin\theta = 0 \quad 4-3\cos\theta = 0$
$\theta = 0, \pi, 2\pi \quad \cos\theta = \frac{4}{3}$
N/A

upper half $0 \le \theta \le \pi$

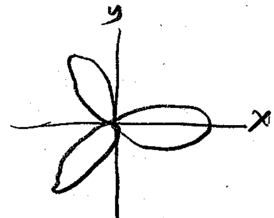
$\boxed{\text{entire figure: } 0 \le \theta \le 2\pi}$

#10. $r = 5$



$\boxed{0 \le \theta \le 2\pi}$

#11. $r = 4\cos(3\theta)$



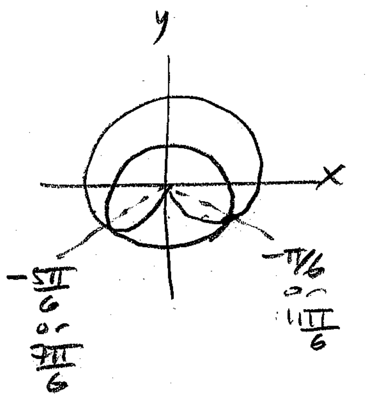
petals start/end when $r=0$
$4\cos(3\theta)=0, \cos(3\theta)=0$
$3\theta = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$
$\theta = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{3\pi}{6} = \frac{\pi}{2}, \frac{5\pi}{6}$
one petal: $-\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$
$\boxed{\text{entire figure: } -\frac{\pi}{6} \le \theta \le \frac{5\pi}{6}}$

For which values of θ do the following curves intersect?

#12. $r = 5 + 4\sin\theta, \quad r = 3$

$5 + 4\sin\theta = 3$
$4\sin\theta = -2$
$\sin\theta = -\frac{2}{4} = -\frac{1}{2}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

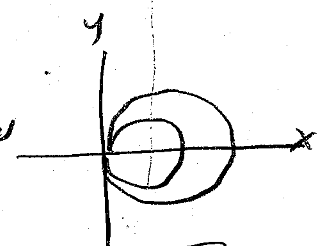


#13. $r = 4\cos\theta, \quad r = 8\cos\theta$

$4\cos\theta = 8\cos\theta$
$4\cos\theta - 8\cos\theta = 0$
$4\cos\theta(1-2) = 0$
$\cos\theta = 0$

$\boxed{\theta = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}}$

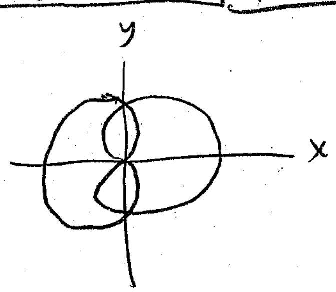
but at these angles
$r = 4\cos(\frac{\pi}{2}) = 0$
$r = 0$
(so this is just the origin)
which is the only intersection



#14. $r = 5(1-\cos\theta), \quad r = 5(1+\cos\theta)$

$5(1-\cos\theta) = 5(1+\cos\theta)$
$1-\cos\theta = 1+\cos\theta$
$2\cos\theta = 0, \quad \cos\theta = 0$

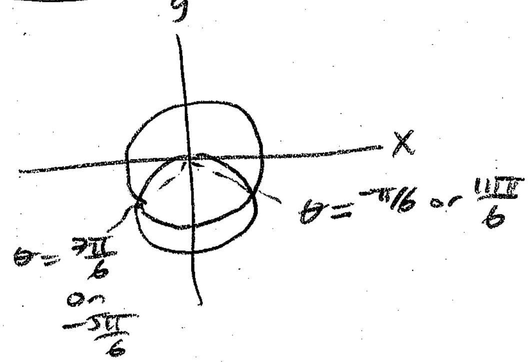
$\boxed{\theta = -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots}$ $\boxed{\text{(plus the origin)}}$



#15. $r = -4\sin\theta, \quad r = 2$

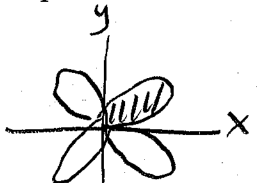
$-4\sin\theta = 2$
$\sin\theta = -\frac{1}{2}$

$\boxed{\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \ldots}$



Find the area described (use calculator to evaluate):

#16. One petal of $r = 4\sin(2\theta)$



start/end $r=0$

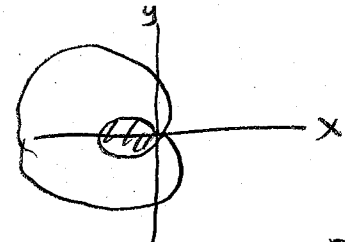
$4\sin(2\theta)=0$, $\sin(2\theta)=0$

$2\theta = 0, \pi, 2\pi, 3\pi$

$\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2} \ldots$

$$\boxed{A = \frac{1}{2}\int_0^{\pi/2} (4\sin(2\theta))^2\, d\theta = 6.283}$$

#17. The inner loop of $r = 2 - 4\cos\theta$



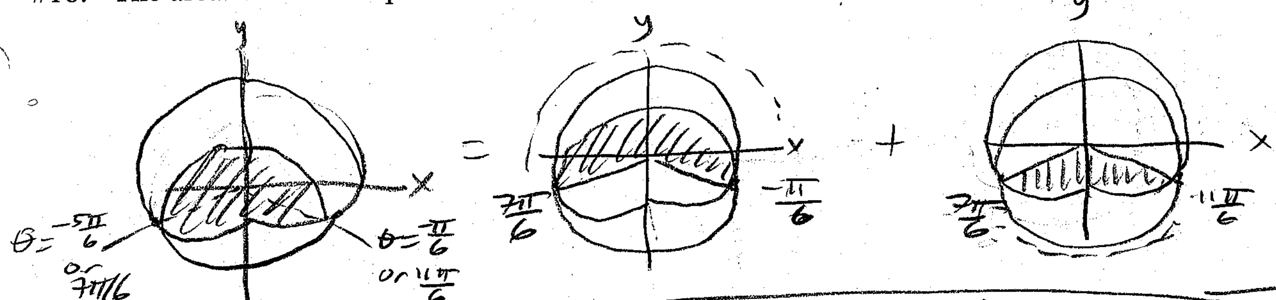
start/end at $r=0$, $2-4\cos\theta=0$,

$\cos\theta = \frac{1}{2}$, $\theta = -\frac{\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

verify (inner) ✓

$$\boxed{A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (2-4\cos\theta)^2\, d\theta = 2.174}$$

#18. The area within both polar curves: $r = 5 + 4\sin\theta$, $r = 3$



intersections:

$5+4\sin\theta = 3$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$

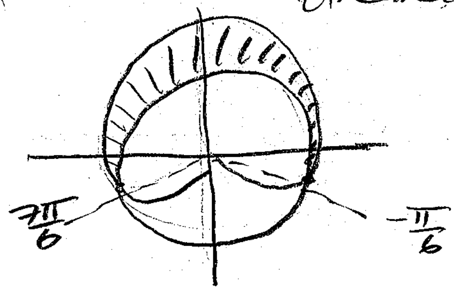
$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$$\boxed{A = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (3)^2\, d\theta + \frac{1}{2}\int_{7\pi/6}^{11\pi/6} (5+4\sin\theta)^2\, d\theta = 22.230}$$

Find the area described (use calculator to evaluate):

#19. The area between the polar curves: $r = 5 + 4\sin\theta, \quad r = 3$

(the area with more positive y)



intersections:

$5+4\sin\theta = 3$

$4\sin\theta = -2$

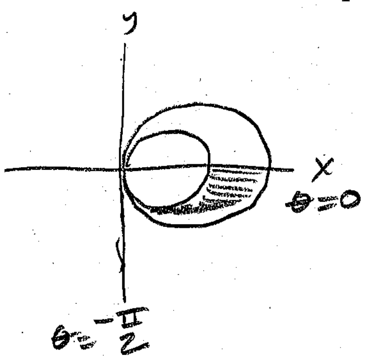
$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$$\boxed{A = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (5+4\sin\theta)^2 d\theta - \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (3)^2 d\theta = 81.442}$$

(outer) − (inner) case

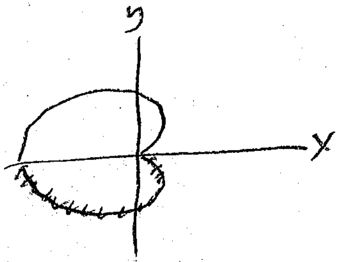
#20. The area between the polar curves and below the x-axis: $r = 4\cos\theta, \quad r = 8\cos\theta$



$$\boxed{A = \frac{1}{2}\int_{-\pi/2}^{0} (8\cos\theta)^2 d\theta - \frac{1}{2}\int_{-\pi/2}^{0} (4\cos\theta)^2 d\theta = 18.850}$$

Find the arc length of the curve (evaluate the integrals by hand):

#21. The part of the cardioid $r = 3 - 3\cos\theta$ which is below the x-axis.



$$\frac{dr}{d\theta} = 3\sin\theta$$

$$\text{arc length} = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\boxed{\text{arc length} = \int_{-\pi}^{0} \sqrt{(3-3\cos\theta)^2 + (3\sin\theta)^2}\, d\theta}$$

start/end at $y=0$

$y = r\sin\theta = (3-3\cos\theta)\sin\theta = 0$

$3 - 3\cos\theta = 0$ $\quad$ $\sin\theta = 0$

$\cos\theta = 1$ $\quad$ $\theta = 0, \pi, -\pi$

$\theta = 0$

Suggest $-\pi \le \theta \le 0$

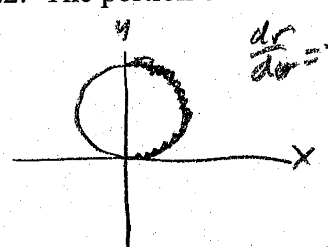
$$= \int_{-\pi}^{0} \sqrt{9 - 18\cos\theta + 9\cos^2\theta + 9\sin^2\theta}\, d\theta$$

$$= \int_{-\pi}^{0} \sqrt{9 - 18\cos\theta + 9(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$= \int_{-\pi}^{0} \sqrt{9 - 18\cos\theta + 9}\, d\theta = \int_{-\pi}^{0} \sqrt{18 - 18\cos\theta}\, d\theta$$

$\boxed{7.12}$ OKAY, still need math 9 here, but on test you'll be able to hand evaluate :)

#22. The portion of $r = 4\sin(\theta)$ with positive x values.



$$\frac{dr}{d\theta} = 4\cos\theta$$

$$\boxed{\text{arc length} = \int_0^{\pi/2} \sqrt{(4\sin\theta)^2 + (-4\cos\theta)^2}\, d\theta}$$

start/end when $x = 0$

$x = r\cos\theta = 4\sin\theta\cos\theta = 0$

$\sin\theta = 0$ $\quad$ $\cos\theta = 0$

$\theta = 0, \pi, 2\pi$ $\quad$ $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

suggests $0 \le \theta \le \frac{\pi}{2}$

✓

$$= \int_0^{\pi/2} \sqrt{16\sin^2\theta + 16\cos^2\theta}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{16(\sin^2\theta + \cos^2\theta)}\, d\theta$$

$$= \int_0^{\pi/2} \sqrt{16}\, d\theta = \int_0^{\pi/2} 4\, d\theta$$

$$= \left[4\theta\right]_0^{\pi/2}$$

$$= 4\left(\frac{\pi}{2}\right) - 4(0) = \boxed{2\pi}$$