AP Calculus BC – Unit 8 Part 3 Extra Practice

8.7 – Extra Practice

Show that the vectors $\vec{u}$ and $\vec{v}$ between the given points are equivalent.

#5b. $\vec{u}: (-4,0)\ to\ (1,8)$

$\vec{v}: (2,-1)\ to\ (7,7)$

$\vec{u} = \langle 1-(-4), 8-0\rangle = \langle 5,8\rangle$

$\vec{v} = \langle 7-2, 7-(-1)\rangle = \langle 5,8\rangle$

Find the magnitude of the vector.

#6b. $\vec{a} = \langle -3,0\rangle$

$|\vec{a}| = \sqrt{(-3)^2+0^2} = \boxed{3}$

#7b. $\vec{a} = \langle 12,-5\rangle$

$|\vec{a}| = \sqrt{12^2+(-5)^2}$
$= \sqrt{12^2+5^2}$
$= \sqrt{169}$
$= \boxed{13}$

#8b. $\vec{a} = \langle -10,3\rangle$

$|\vec{a}| = \sqrt{10^2+3^2}$
$= \boxed{\sqrt{109}}$

For the given vectors, find the new vectors: $\frac{2}{3}\vec{a},\quad 3\vec{b},\quad \vec{b}-\vec{a},\quad 2\vec{a}+5\vec{b}$.

#9b. $\vec{a} = \langle -3,-8\rangle$

$\vec{b} = \langle 8,25\rangle$

$\frac{2}{3}\vec{a} = \frac{2}{3}\langle -3,-8\rangle = \langle \frac{2}{3}(-3), \frac{2}{3}(-8)\rangle = \boxed{\langle -2, -\frac{16}{3}\rangle}$

$3\vec{b} = 3\langle 8,25\rangle = \boxed{\langle 24,75\rangle}$

$\vec{b}-\vec{a} = \langle 8,25\rangle - \langle -3,-8\rangle = \langle 8-(-3), 25-(-8)\rangle = \boxed{\langle 11,33\rangle}$

$2\vec{a}+5\vec{b} = 2\langle -3,-8\rangle + 5\langle 8,25\rangle$
$= \langle -6,-16\rangle + \langle 40,125\rangle$
$= \boxed{\langle 34,109\rangle}$

For the given vector, find: $\left|\vec{a}\right|$, $\dfrac{\vec{a}}{\left|\vec{a}\right|}$, $\left|\dfrac{\vec{a}}{\left|\vec{a}\right|}\right|$.

#10b. $\vec{a} = \langle 4, -7 \rangle$

$$\boxed{|\vec{a}| = \sqrt{4^2 + 7^2} = \sqrt{65}}$$

$$\boxed{\frac{\vec{a}}{|\vec{a}|} = \frac{\langle 4, -7 \rangle}{\sqrt{65}} = \frac{1}{\sqrt{65}} \langle 4, -7 \rangle = \left\langle \frac{4}{\sqrt{65}}, \frac{-7}{\sqrt{65}} \right\rangle}$$

$$\boxed{\left|\frac{\vec{a}}{|\vec{a}|}\right| = \sqrt{\left(\frac{4}{\sqrt{65}}\right)^2 + \left(\frac{-7}{\sqrt{65}}\right)^2} = 1}$$

Convert the given vector into its x- and y-components.

#11b. $\left|\vec{a}\right| = 2$, $\theta = 150°$

$$\vec{a} = \langle |\vec{a}| \cos(\theta_a), |\vec{a}| \sin(\theta_a) \rangle$$

$$\vec{a} = \langle 2 \cos(150°), 2 \sin(150°) \rangle$$

$$\vec{a} = \left\langle 2\left(-\frac{\sqrt{3}}{2}\right), 2\left(\frac{1}{2}\right) \right\rangle$$

$$\boxed{\vec{a} = \langle -\sqrt{3}, 1 \rangle}$$

8.8 – Extra Practice

Find the domain of the vector-valued function.

#4b. $\vec{r}(t) = \langle \sqrt{4-t^2},\ t^2 \rangle$

$4 - t^2 = 0$

$t = \pm 2$

$-\infty \quad -2 \quad 2 \quad \infty$

x ✓ ✓ ✓ x

$\boxed{D: [-2, 2]}$

#5b. $\vec{r}(t) = \langle \sin(t),\ \cos(t) \rangle$

no restrictions

$\boxed{D: (-\infty, \infty)}$

For the vector-valued function $\vec{r}(t) = \left\langle \frac{1}{2}t^2,\ -t+1 \right\rangle$ evaluate at its indicated input value.

#6b. $\vec{r}(s+1)$

$\boxed{\vec{r}(s+1) = \langle \tfrac{1}{2}(s+1)^2,\ -(s+1)+1 \rangle}$

#7b. $\vec{r}(2+\Delta t) - \vec{r}(2)$

$\langle \tfrac{1}{2}(2+\Delta t)^2,\ -(2+\Delta t)+1 \rangle - \langle \tfrac{1}{2}(2)^2,\ -(2)+1 \rangle$

$\langle \tfrac{1}{2}(2+\Delta t)^2,\ -2-\Delta t+1 \rangle - \langle 2, -1 \rangle$

$\boxed{\langle \tfrac{1}{2}(2+\Delta t)^2 - 2,\ -\Delta t \rangle}$

Sketch the plane curve for the given vector-valued function (with direction arrows).

#8b. $\vec{r}(t) = \langle 5-t,\ \sqrt{t} \rangle$

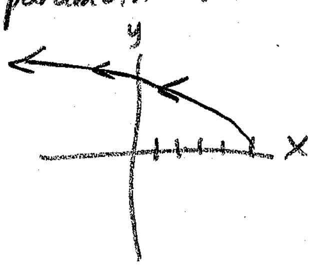
table

t	$\langle 5-t, \sqrt{t} \rangle$
0	$\langle 5, 0 \rangle$
1	$\langle 4, 1 \rangle$
4	$\langle 1, 2 \rangle$
⋮	

or convert parametric

$\begin{cases} x = 5-t \\ y = \sqrt{t} \end{cases} \quad t = 5-x \quad y = \sqrt{5-x}$

or calculator in parametric mode:

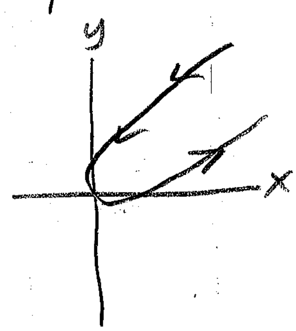


#9b. $\vec{r}(t) = \langle t^2+t,\ t^2-t \rangle$

table

t	$\langle t^2+t, t^2-t \rangle$
−2	$\langle 6, 6 \rangle$
−1	$\langle 2, 0 \rangle$
0	$\langle 0, 0 \rangle$
1	$\langle 2, 0 \rangle$
2	$\langle 6, 2 \rangle$
⋮	

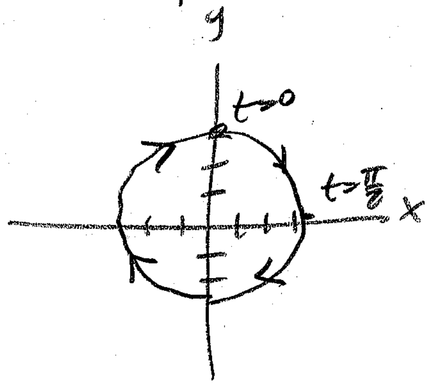
or calculator in parametric mode:



Sketch the plane curve for the given vector-valued function (with direction arrows).

#10b. $\vec{r}(t) = \langle 3\sin(t),\ 3\cos(t) \rangle$

This is parametric form for circle w/ r=3



direction?

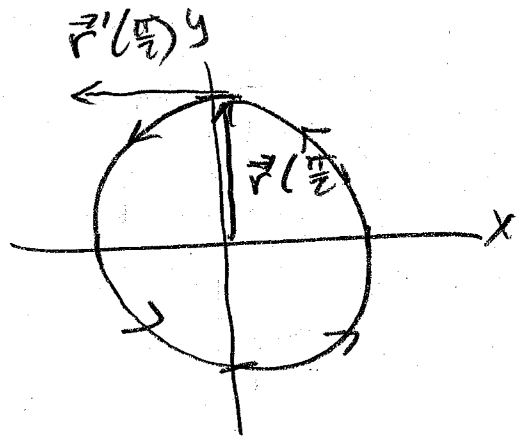
t	$\langle 3\sin t, 3\cos t \rangle$
0	$\langle 0, 3 \rangle$
$\frac{\pi}{2}$	$\langle 3, 0 \rangle$

Find $\vec{r}'(t)$ then $\vec{r}\left(\frac{\pi}{2}\right)$ and $\vec{r}'\left(\frac{\pi}{2}\right)$. Then sketch the plane curve, and add the vectors to your sketch $\vec{r}\left(\frac{\pi}{2}\right)$ and $\vec{r}'\left(\frac{\pi}{2}\right)$.

#11b. $\vec{r}(t) = \langle \cos(t),\ \sin(t) \rangle$

$$\boxed{\vec{r}'(t) = \langle -\sin t, \cos t \rangle}$$

$$\boxed{\begin{aligned} \vec{r}\left(\tfrac{\pi}{2}\right) &= \left\langle \cos\left(\tfrac{\pi}{2}\right), \sin\left(\tfrac{\pi}{2}\right) \right\rangle = \langle 0, 1 \rangle \\ \vec{r}'\left(\tfrac{\pi}{2}\right) &= \left\langle -\sin\left(\tfrac{\pi}{2}\right), \cos\left(\tfrac{\pi}{2}\right) \right\rangle = \langle -1, 0 \rangle \end{aligned}}$$



$\vec{r}$ always drawn from origin to point on curve

$\vec{r}'$ always drawn from the point and should be tangent to the curve in the direction of motion

8.9 – Extra Practice

Position vector $\vec{r}(t)$ represents the path of an object moving on a plane.

(a) Find $\vec{v}(t)$, $\vec{a}(t)$ and the speed of the object at time *t*.

(b) Evaluate the position, velocity, and acceleration vectors at the given time.

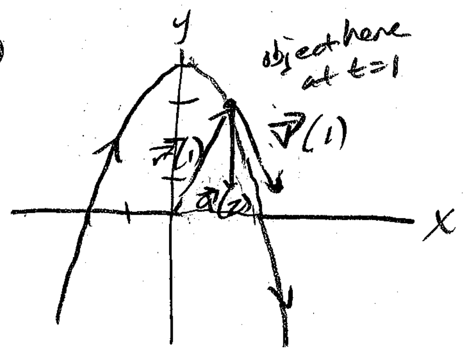
(c) Sketch the path of the object and add the position, velocity, and acceleration vectors at the given time to your sketch.

#4b. $\vec{r}(t)=\langle t, -t^2+4\rangle$ at $t=1$

(a) $\boxed{\begin{array}{l}\vec{v}(t)=\langle 1,-2t\rangle \\ \vec{a}(t)=\langle 0,-2\rangle \\ \text{speed}=|\vec{v}|=\sqrt{1^2+(-2t)^2}\end{array}}$

(b) $\boxed{\begin{array}{l}\vec{r}(1)=\langle 1,3\rangle \\ \vec{v}(1)=\langle 1,-2\rangle \\ \vec{a}(1)=\langle 0,-2\rangle\end{array}}$

(c)



t	$\langle t,-t^2+4\rangle$
0	$\langle 0,4\rangle$
1	$\langle 1,3\rangle$

position vectors from origin to object

velocity & acceleration start at the object

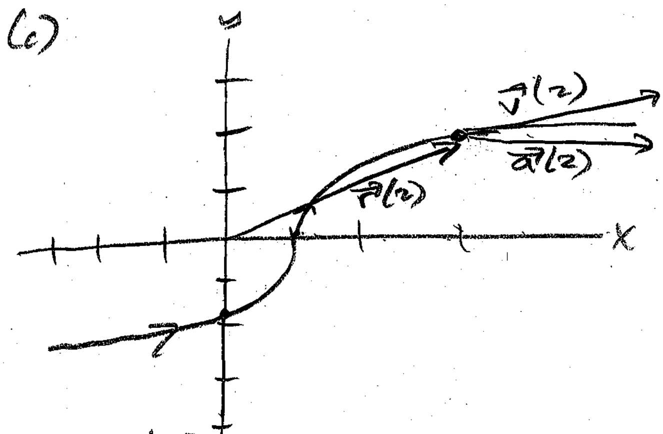
(use calculator for plot)

#5b. $\vec{r}(t)=\left\langle \frac{1}{4}t^3+1,\ t\right\rangle$ at $t=2$

(a) $\boxed{\begin{array}{l}\vec{v}(t)=\langle \frac{3}{4}t^2,1\rangle \\ \vec{a}(t)=\langle \frac{3}{2}t,0\rangle \\ \text{speed}=|\vec{v}|=\sqrt{(\frac{3}{4}(t^2))^2+1^2}\end{array}}$

(b) $\boxed{\begin{array}{l}\vec{r}(2)=\langle 3,2\rangle \\ \vec{v}(2)=\langle 3,1\rangle \\ \vec{a}(2)=\langle 3,0\rangle\end{array}}$

(c)



t	$\langle \frac{1}{4}t^3+1,t\rangle$
0	$\langle 1,0\rangle$
1	$\langle \frac{5}{4},1\rangle$

Position vector $\vec{r}(t)$ represents the path of an object moving on a plane.

(a) Find $\vec{v}(t)$, $\vec{a}(t)$ and the speed of the object at time t.

(b) Evaluate the position, velocity, and acceleration vectors at the given time.

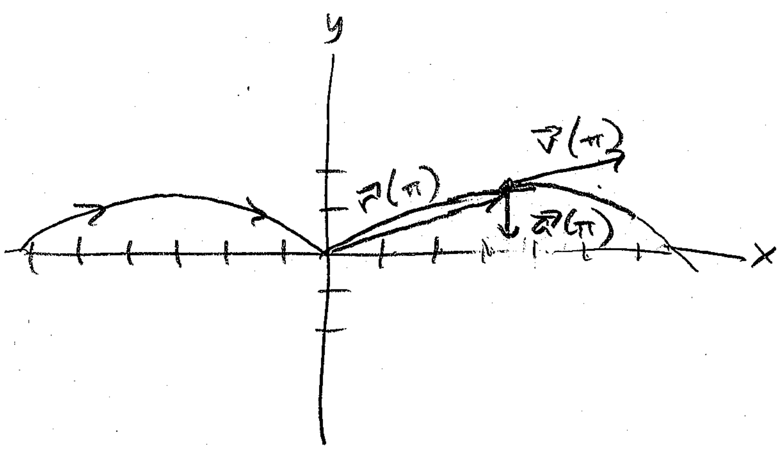
(c) Sketch the path of the object and add the position, velocity, and acceleration vectors at the given time to your sketch.

#6b. $\vec{r}(t)=\langle t-\sin(t),\ 1-\cos(t)\rangle$ at $t=\pi$

(a)
$$\boxed{\begin{aligned}&\vec v(t) = \langle 1-\cos t,\ 1+\sin t\rangle\\ &\vec a(t) = \langle \sin t,\ \cos t\rangle\\ &\text{speed} = |\vec v| = \sqrt{(1-\cos t)^2+(1+\sin t)^2}\end{aligned}}$$

(b)
$$\boxed{\begin{aligned}\vec r(\pi) &= \langle \pi-\sin\pi,\ 1-\cos\pi\rangle\\ &= \langle \pi,\ 2\rangle\\ \vec v(\pi) &= \langle 1-\cos\pi,\ 1+\sin\pi\rangle\\ &= \langle 2,\ 1\rangle\\ \vec a(\pi) &= \langle \sin\pi,\ \cos\pi\rangle\\ &= \langle 0,\ -1\rangle\end{aligned}}$$

(c)



The velocity vector and the position of a particle at time $t = 0$ are given.
(a) Find the position of the particle at time $t = 3$.
(b) Find the total distance traveled by the particle from $0 \le t \le 3$.
(c) Find the position vector for any time t.
(d) Find the displacement of the particle from $0 \le t \le 3$.

#7b. $\vec{v}(t) = \langle 8t - 1,\ 6t^2 + 1 \rangle$, $\vec{r}(0) = \langle 4, 0 \rangle$

(a) $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 4t^2 + C,\ 2t^3 + D \rangle$

$\vec{r}(0) = \langle 4, 0 \rangle = \langle 4(0)^2 + C,\ 2(0)^3 + D \rangle = \langle C, D \rangle$ $C = 4,\ D = 0$

$\vec{r}(t) = \langle 4t^2 + 4,\ 2t^3 \rangle$

$\vec{r}(3) = \langle 4(3)^2 + 4,\ 2(3)^3 \rangle = \boxed{\langle 40, 54 \rangle}$

(b) total dist $= \int_0^3 |\vec{v}(t)|\,dt = \boxed{\int_0^3 \sqrt{(8t-1)^2 + (6t^2+1)^2}\,dt = 66.429}$

(c) from part a, $\boxed{\vec{r}(t) = \langle 4t^2 + 4,\ 2t^3 \rangle}$

(d) displacement $= \vec{r}(3) - \vec{r}(0)$

$= \langle 40, 54 \rangle - \langle 4, 0 \rangle$

$= \boxed{\langle 36, 54 \rangle}$

Use the given information to find the velocity and position vectors. Then find the position at time $t = 2$.

#8b. $\vec{a}(t) = \langle t, \sin(t) \rangle$, $\vec{v}(0) = \langle 0, -1 \rangle$, $\vec{r}(0) = \langle 0, 0 \rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle \frac{1}{2}t^2 + C, -\cos t + D \rangle$

$\vec{v}(0) = \langle 0, -1 \rangle = \langle \frac{1}{2}(0)^2 + C, \underbrace{-\cos(0) + D}_{-1+D} \rangle$ $C = 0$, $-1 + D = -1$, $D = 0$

$\boxed{\vec{v}(t) = \langle \frac{1}{2}t^2, -\cos t \rangle}$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{1}{6}t^3 + E, -\sin t + F \rangle$

$\vec{r}(0) = \langle 0, 0 \rangle = \langle \frac{1}{6}(0)^3 + E, -\sin(0) + F \rangle = \langle E, F \rangle$ $E = 0$, $F = 0$

$\boxed{\vec{r}(t) = \langle \frac{1}{6}t^3, -\sin t \rangle}$

$\vec{r}(2) = \langle \frac{1}{6}(2)^3, -\sin(2) \rangle = \boxed{\langle \frac{4}{3}, -\sin(2) \rangle}$