AP Calculus BC – Unit 8 Part 3 Required Practice Name: answers only

8.7 – Required Practice

#1. Show vectors are equivalent:

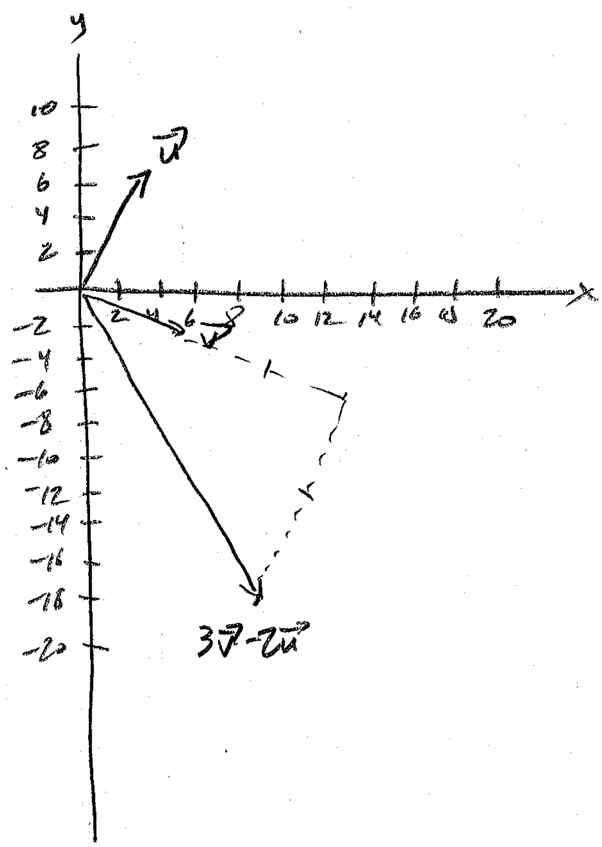
u: (−4, 0), (1, 8)
v: (2, −1), (7, 7)

both are $\langle 5,8 \rangle$

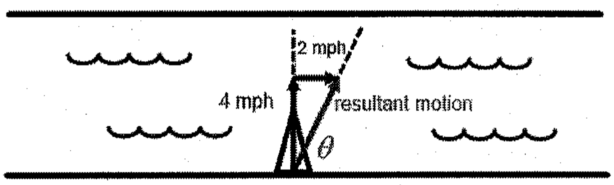
#2. Find the magnitude of each vector and 3v - 2u
Sketch the original vectors and 3v - 2u

$\vec{u} = \langle 3, 6 \rangle$
$\vec{v} = \langle 5, -2 \rangle$

$|\vec{u}| = \sqrt{45}$
$|\vec{v}| = \sqrt{29}$
$3\vec{v} - 2\vec{u} = \langle 9, -18 \rangle$
$|3\vec{v} - 2\vec{u}| = \sqrt{405}$



#3. A boat heads straight across a river at a speed of 4 mph, but the water in the river is flowing a 2 mph (as in the figure). What is the resultant and direction of the boat?

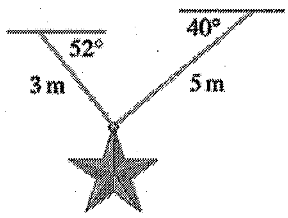


$\boxed{\vec{r} = \langle 2, 4 \rangle}$

$\boxed{|\vec{r}| = \sqrt{20} = 4.472 \text{ mph}}$

$\boxed{\theta_r = 63.434^\circ}$

#4. A star-shaped decoration is suspended, motionless, from two points as shown in the figure. If the decoration weighs 10 lbs, find the tension in each wire (the magnitude of each tension as well as the component forces)?



$|\vec{T_1}| = 7.665$ lbs

$|T_2| = 6.160$ lbs

$\vec{T_1} = \langle -4.719, 6.040 \rangle$ (lbs)

$\vec{T_2} = \langle 4.719, 3.960 \rangle$ (lbs)

Show that the vectors $\vec{u}$ and $\vec{v}$ between the given points are equivalent.

#5. $\vec{u}: (3,2)\ to\ (5,6)$

$\vec{v}: (1,4)\ to\ (3,8)$

$\boxed{\text{both are } \langle 2,4 \rangle}$

Find the magnitude of the vector.

#6. $\vec{a} = \langle 7,0 \rangle$

$\boxed{|\vec{a}| = 7}$

#7. $\vec{a} = \langle 4,3 \rangle$

$\boxed{|\vec{a}| = 5}$

#8. $\vec{a} = \langle 6,-5 \rangle$

$\boxed{|\vec{a}| = \sqrt{61}}$

For the given vectors, find the new vectors: $\frac{2}{3}\vec{a},\quad 3\vec{b},\quad \vec{b}-\vec{a},\quad 2\vec{a}+5\vec{b}$.

#9. $\vec{a} = \langle 4,9 \rangle$

$\vec{b} = \langle 2,-5 \rangle$

$\frac{2}{3}\vec{a} = \langle \frac{8}{3}, 6 \rangle$

$3\vec{b} = \langle 6, -15 \rangle$

$\vec{b} - \vec{a} = \langle -2, -14 \rangle$

$2\vec{a} + 5\vec{b} = \langle 18, -7 \rangle$

For the given vector, find: $\left|\vec{a}\right|$, $\dfrac{\vec{a}}{\left|\vec{a}\right|}$, $\left|\dfrac{\vec{a}}{\left|\vec{a}\right|}\right|$.

#10. $\vec{a} = \langle 3, -4 \rangle$

$$\boxed{\begin{aligned} &|\vec{a}| = 5 \\ &\frac{\vec{a}}{|\vec{a}|} = \frac{1}{5}\langle 3, -4\rangle = \left\langle \frac{3}{5}, \frac{-4}{5} \right\rangle \\ &\left|\frac{\vec{a}}{|\vec{a}|}\right| = 1 \end{aligned}}$$

Convert the given vector into its x- and y-components.

#11. $\left|\vec{a}\right| = 3, \quad \theta = 60°$

$$\boxed{\vec{a} = \left\langle \frac{3}{2}, \frac{3\sqrt{3}}{2} \right\rangle}$$

8.8 – Required Practice

#1. Find the domain of the vector-valued function: $\vec{r}(t)=\left\langle \sqrt{4-t^2}, \frac{3}{t-1} \right\rangle$.

$\boxed{\text{Domain: } [-2,1)\cup(1,2]}$

#2. Find the derivative of $\vec{r}(t)=\left\langle e^{(t^2+3t)}, \frac{\ln(t^2-3t)}{t^3-4} \right\rangle$

$$\boxed{\vec{r}\,'(t)=\left< e^{(t^2+3t)}(2t+3),\ \frac{(t^3-4)\frac{1}{t^2-3t}(2t-3)-\ln(t^2-3t)(3t^2)}{(t^3-4)^2} \right>}$$

#3. Evaluate $\int \vec{r}(t)\,dt$ if $\vec{r}(t)=\langle \cos t,\ t^3-t \rangle$

$$\int \vec{r}(t)\,dt = \left< \sin t + C,\ \tfrac{1}{4}t^4-\tfrac{1}{2}t^2+D \right>$$

or $\left< \sin t,\ \tfrac{1}{4}t^4-\tfrac{1}{2}t^2 \right>+\vec{C}$

Find the domain of the vector-valued function.

#4. $\vec{r}(t)=\left\langle \frac{1}{t+1}, \frac{t}{2} \right\rangle$

$\boxed{D: (-\infty,-1)\cup(-1,\infty)}$

#5. $\vec{r}(t)=\left\langle \ln(t), -e^{t} \right\rangle$

$\boxed{D: (0,\infty)}$

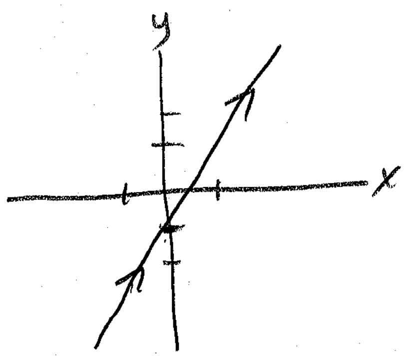
For the vector-valued function $\vec{r}(t)=\left\langle \frac{1}{2}t^{2}, -t+1 \right\rangle$ evaluate at its indicated input value.

#6. $\boxed{\vec{r}(1) = \langle \frac{1}{2}, 0 \rangle}$

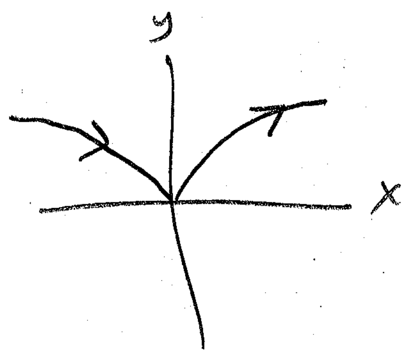
#7. $\boxed{\vec{r}(2+\Delta t) = \langle \frac{1}{2}(2+\Delta t)^2, -(2+\Delta t)+1 \rangle}$

Sketch the plane curve for the given vector-valued function (with direction arrows).

#8. $\vec{r}(t)=\left\langle \frac{1}{4}t, t-1 \right\rangle$



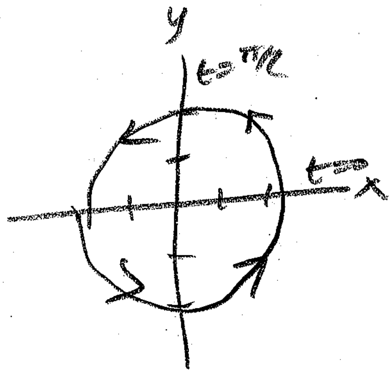
#9. $\vec{r}(t)=\left\langle t^{3}, t^{2} \right\rangle$



Sketch the plane curve for the given vector-valued function (with direction arrows).

#10. $\vec{r}(t) = \langle 2\cos(t),\ 2\sin(t) \rangle$

this is parametric form for circle $r=2$;



t	$\langle 2\cos t, 2\sin t \rangle$
0	$\langle 2, 0 \rangle$
$\frac{\pi}{2}$	$\langle 0, 2 \rangle$

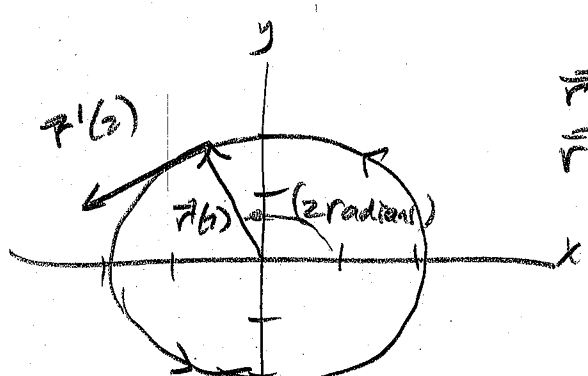
Find $\vec{r}'(t)$ then $\vec{r}(2)$ and $\vec{r}'(2)$. Then sketch the plane curve, and add the vectors to your sketch $\vec{r}(2)$ and $\vec{r}'(2)$.

#11. $\vec{r}(t) = \langle 2\cos(t),\ 2\sin(t) \rangle$

$\boxed{\vec{r}'(t) = \langle -2\sin t,\ 2\cos t \rangle}$

$\boxed{\vec{r}(2) = \langle 2\cos(2), 2\sin(2) \rangle = \langle -0.832, 1.819 \rangle}$

$\boxed{\vec{r}'(2) = \langle -2\sin(2), 2\cos(2) \rangle = \langle -1.819, -0.832 \rangle}$



$\vec{r}$ always from origin to point on curve
$\vec{r}'$ is drawn from the point
(should be tangent to curve in direction of motion)

8.9 – Required Practice

#1. $\vec{r}(t)=\langle t^3+2\cos t,\ \sin t-t^2\rangle$

Find $\vec{v}(t), \vec{a}(t)$

$\vec{v}(t) = \langle 3t^2-2\sin t,\ \cos t-2t\rangle$

$\vec{a}(t) = \langle 6t-2\cos t,\ -\sin t-2\rangle$

#2. $\vec{a}(t)=\langle t, -10\rangle, \quad \vec{v}(2)=\langle 5,2\rangle, \quad \vec{r}(1)=\langle 3,4\rangle$

Find $\vec{v}(t), \vec{r}(t), \vec{r}(3)$

$\vec{v}(t) = \langle \frac{1}{2}t^2+3,\ -10t+22\rangle$

$\vec{r}(t) = \langle \frac{1}{6}t^3+3t-\frac{1}{6},\ -5t^2+22t-13\rangle$

$\vec{r}(3) = \langle \frac{40}{3},\ 8\rangle$

#3. $\vec{a}(t)=\langle 2, 2t\rangle$, $\vec{v}(1)=\langle 3,5\rangle$, $\vec{r}(1)=\langle 2,6\rangle$

Find $\vec{v}(t)$, $\vec{r}(t)$, $\vec{r}(2)$ **and total distance traveled from $t=0$ to $t=2$**

$\vec{v}(t) = \langle 2t+1, t^2+4\rangle$

$\vec{r}(t) = \langle t^2+t, \frac{1}{3}t^3+4t+\frac{5}{3}\rangle$

$\vec{r}(2) = \langle 6, \frac{37}{3}\rangle$

total dist $= \int_0^2 \sqrt{(2t+1)^2+(t^2+4)^2}\, dt = 12.281$

Position vector $\vec{r}(t)$ represents the path of an object moving on a plane.

(a) Find $\vec{v}(t)$, $\vec{a}(t)$ and the speed of the object at time t.

(b) Evaluate the position, velocity, and acceleration vectors at the given time.

(c) Sketch the path of the object and add the position, velocity, and acceleration vectors at the given time to your sketch.

#4. $\vec{r}(t) = \langle 3t,\ t-1 \rangle$ at $t=1$

(a) $\vec{v}(t) = \langle 3, 1 \rangle$

$\vec{a}(t) = \langle 0, 0 \rangle$

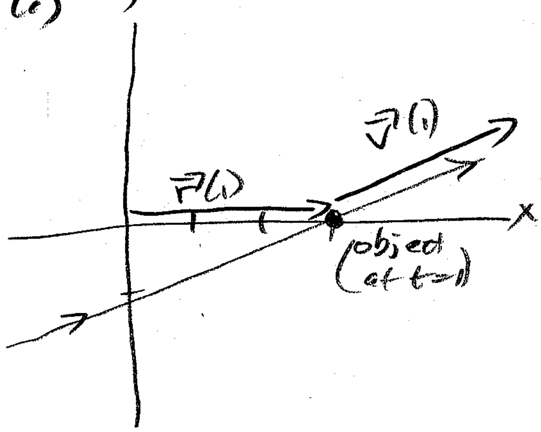
speed $= \sqrt{10}$

(b) $\vec{r}(1) = \langle 3, 0 \rangle$

$\vec{v}(1) = \langle 3, 1 \rangle$

$\vec{a}(1) = \langle 0, 0 \rangle$

(c)



#5. $\vec{r}(t) = \langle t^2,\ t \rangle$ at $t=2$

(a) $\vec{v}(t) = \langle 2t, 1 \rangle$

$\vec{a}(t) = \langle 2, 0 \rangle$

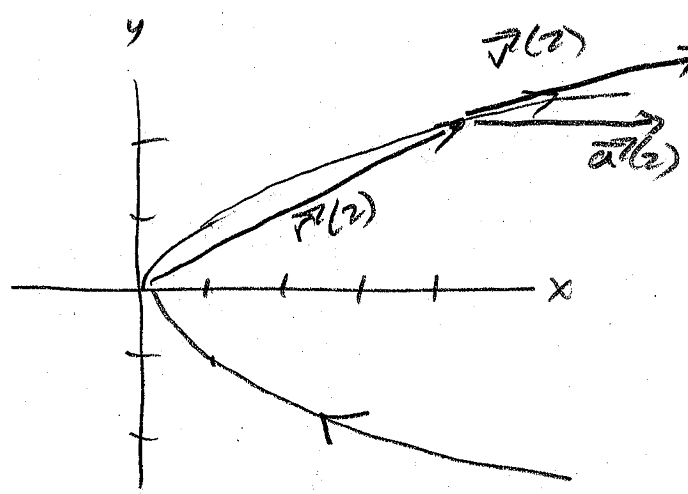
speed $= \sqrt{4t^2+1}$

(b) $\vec{r}(2) = \langle 4, 2 \rangle$

$\vec{v}(2) = \langle 4, 1 \rangle$

$\vec{a}(2) = \langle 2, 0 \rangle$

(c)



Position vector $\vec{r}(t)$ represents the path of an object moving on a plane.

(a) Find $\vec{v}(t)$, $\vec{a}(t)$ and the speed of the object at time *t*.

(b) Evaluate the position, velocity, and acceleration vectors at the given time.

(c) Sketch the path of the object and add the position, velocity, and acceleration vectors at the given time to your sketch.

#6. $\vec{r}(t)=\langle 2\cos(t),\ 2\sin(t)\rangle$ at $t=\frac{\pi}{4}$

(a) $\vec{v}(t) = \langle -2\sin t,\ 2\cos t \rangle$

$\vec{a}(t) = \langle -2\cos t, -2\sin t \rangle$

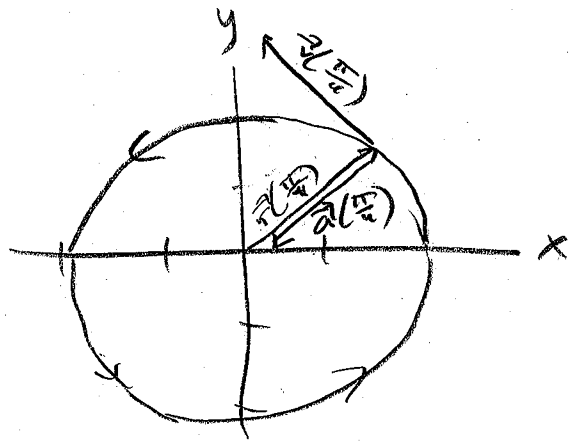
Speed $= 2$

(b) $\vec{r}(\frac{\pi}{4}) = \langle \sqrt{2}, \sqrt{2} \rangle$

$\vec{v}(\frac{\pi}{4}) = \langle -\sqrt{2}, \sqrt{2} \rangle$

$\vec{a}(\frac{\pi}{4}) = \langle -\sqrt{2}, -\sqrt{2} \rangle$

(c)



The velocity vector and the position of a particle at time $t = 0$ are given.
(a) Find the position of the particle at time $t = 3$.
(b) Find the total distance traveled by the particle from $0 \le t \le 3$.
(c) Find the position vector for any time t.
(d) Find the displacement of the particle from $0 \le t \le 3$.

#7. $\vec{v}(t) = \langle 3t^2,\ 2t \rangle$, $\vec{r}(0) = \langle 1, 2 \rangle$

(a) $\boxed{(28, 11)}$

(b) $\boxed{\int_0^3 |\vec{v}(t)|\, dt = 28.728}$

(c) $\boxed{\vec{r}(t) = \langle t^3+1,\ t^2+2 \rangle}$

(d) $\boxed{\text{displacement} = \langle 27, 9 \rangle}$

Use the given information to find the velocity and position vectors. Then find the position at time $t = 2$.

#8. $\vec{a}(t)=\langle 4t,\ t^2\rangle,\quad \vec{v}(0)=\langle 5,0\rangle,\quad \vec{r}(0)=\langle 4,2\rangle$

$$\boxed{\begin{aligned}&\vec{v}(t) = \langle 2t^2+5,\ \tfrac{1}{3}t^3\rangle\\ &\vec{r}(t) = \langle \tfrac{2}{3}t^3+5t+4,\ \tfrac{1}{12}t^4+2\rangle\\ &\vec{r}(2) = \langle \tfrac{58}{3}, 6\rangle \text{ or } \left(\tfrac{58}{3}, 6\right)\end{aligned}}$$

Unit 8 Part 3 Test Review

Given vectors $\vec{a}=\langle -3, 6\rangle$, $\vec{b}=\langle 2,-3\rangle$, $\vec{c}=\langle 1,5\rangle$

#1. Find $2\vec{a}-3\vec{b}+5\vec{c}$

$2\langle -3,6\rangle-3\langle 2,-3\rangle+5\langle 1,5\rangle$

$\langle -6,12\rangle-\langle 6,-9\rangle+\langle 5,25\rangle$

$\boxed{\langle -7, 46\rangle}$

#2. Find $|\vec{b}|$

$|\vec{b}|=\sqrt{2^2+3^2}=\boxed{\sqrt{13}}$

Given position vector $\vec{r}(t)=\langle 5t^2-t,\ e^t+\sin(2t)\rangle$

#3. Find $\vec{r}(-2)$

$\vec{r}(-2)=\langle 5(-2)^2-(-2),\ e^{-2}+\sin(2(-2))\rangle$

$=\boxed{\langle 22,\ e^{-2}+\sin(-4)\rangle}$

#4. Find $\vec{r}(2m-3n)$

$\boxed{\vec{r}(2m-3n)= \langle 5(2m-3n)^2-(2m-3n),\ e^{2m-3n}+\sin(2(2m-3n$

For the position vector $\vec{r}(t)=\langle 3\sin(t),\ 2\cos(t)\rangle$

#4. Find $\vec{v}(t)$

$\boxed{\vec{v}(t)=\langle 3\cos t, -2\sin t\rangle}$

#5. Find $\vec{r}\left(\frac{2\pi}{3}\right)$ and $\vec{v}\left(\frac{2\pi}{3}\right)$

$\vec{r}\left(\frac{2\pi}{3}\right)=\langle 3\sin\left(\frac{2\pi}{3}\right), 2\cos\left(\frac{2\pi}{3}\right)\rangle$

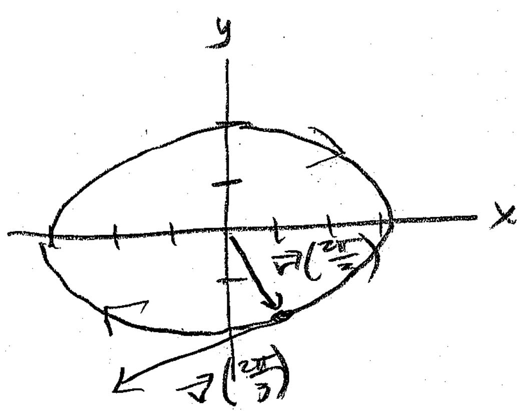
$=\langle 3\frac{\sqrt{3}}{2}, 2\left(-\frac{1}{2}\right)\rangle=\boxed{\langle \frac{3\sqrt{3}}{2}, -1\rangle}$

$\vec{v}\left(\frac{2\pi}{3}\right)=\langle 3\cos\left(\frac{2\pi}{3}\right), -2\sin\left(\frac{2\pi}{3}\right)\rangle$

$=\langle 3\left(-\frac{1}{2}\right), -2\left(\frac{\sqrt{3}}{2}\right)\rangle=\boxed{\langle -\frac{3}{2}, -\sqrt{3}\rangle}$

#6. Sketch the plane curve for $\vec{r}(t)$ and add to your sketch the vectors $\vec{r}\left(\frac{2\pi}{3}\right)$ and $\vec{v}\left(\frac{2\pi}{3}\right)$

t	$\langle 3\sin t, 2\cos t\rangle$
0	$\langle 0,2\rangle$
$\frac{\pi}{2}$	$\langle 3,0\rangle$



#7. An object is moving in the xy-plane with a velocity vector given by $\vec{v}(t)=\langle 3t^2-4t,\ 2t-4\rangle$. The position of the object at time $t=1$ is $(0,\ -3)$. In this problem, distance is in meters and time is in seconds.

(a) Find the position of the object at time $t=2$

(b) Find the acceleration vector of the object as a function of t.

(c) For what time(s), $-1<t<3$ is the tangent line to the object's path horizontal?

(d) For what time(s), $-1<t<3$ is the tangent line to the object's path vertical?

(e) For what time(s), $-1<t<3$ is the object at rest?

(f) For what time(s), $-1<t<3$ is the tangent line to the object's path have a slope of -2 ?

(g) For what time(s), $-1<t<3$ is the speed of the object $4\frac{m}{s}$?

(h) At $t=1$ is the object moving up or down? Explain.

(i) At $t=1$ is the object moving right or left? Explain.

#7

(a) $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle t^3 - 2t^2 + C,\ t^2 - 4t + D \rangle$

$\vec{r}(1) = \langle 0, -3 \rangle = \langle (1)^3 - 2(1)^2 + C,\ (1)^2 - 4(1) + D \rangle = \langle -1 + C,\ -3 + D \rangle$

$C = 1,\ D = 0$

$\vec{r}(t) = \langle t^3 - 2t^2 + 1,\ t^2 - 4t \rangle$

$\vec{r}(2) = \langle (2)^3 - 2(2)^2 + 1,\ (2)^2 - 4(2) \rangle = \boxed{\langle 1, -4 \rangle}$

(b) $\boxed{\vec{a}(t) = \langle 6t - 4,\ 2 \rangle}$

(c) horizontal tangent when $\frac{dy}{dt} = 0$

$2t - 4 = 0$

$2t = 4$

$\boxed{t = 2 \text{ seconds}}$

(d) vertical tangent when $\frac{dx}{dt} = 0$

$3t^2 - 4t = 0$

$t(3t - 4) = 0 \qquad 3t - 4 = 0,\ t = 4/3$

$\boxed{\begin{array}{c}\text{at } t = 0 \text{ seconds} \\ \text{and } t = 4/3 \text{ seconds}\end{array}}$

(e) At rest requires both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ to be zero at the same time. This $\boxed{\text{does not occur}}$ for $-1 < t < 3$.

(f) tangent slope $= \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{2t-4}{3t^2-4t} = -2$

by calculator graph, this occurs at $\boxed{t = -0.457 \text{ sec}}$ and $\boxed{t = 1.457 \text{ sec}}$

(g) speed $= |v(t)|$, so $\sqrt{(3t^2-4t)^2+(2t-4)^2} = 4$

by calculator graph, this occurs at $\boxed{t = 0 \text{ sec}}$ and $\boxed{t = 2 \text{ sec}}$

(h) At $t=1$ $\vec{v}(1) = \langle 3(1)^2-4(1),\ 2(1)-4 \rangle$

$\vec{v}(1) = \langle -1, -2 \rangle$

Since $\frac{dy}{dt} = -2 < 0$, the object is moving $\boxed{\text{down}}$ at $t=1$

(i) $\vec{v}(1) = \langle -1, -2 \rangle$

Since $\frac{dx}{dt} = -1 < 0$, the object is moving $\boxed{\text{left}}$ at $t=1$