AP Calculus BC Semester 1 Final Exam Review **Name** Solutions

Free-Response Part: You'll need to show work on the final using appropriate formats and explanations. The FRQ part of the final exam is NO CALCULATOR, but the first review problem does require a calculator (do the remaining FRQ review problems without a calculator). Look at the posted solutions to see the required justifications and formatting for full credit.

#1. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{1}{2}t^2\right)$. At time $t = 0$, the particle is at position $x = 1$.

a) Find the acceleration of the particle at time $t = 2$. Is the speed of the particle increasing at $t = 2$? Explain your reasoning.

b) Find all time t in the open interval $0 < t < 3$ when the particle changes direction. Justify your answer.

c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

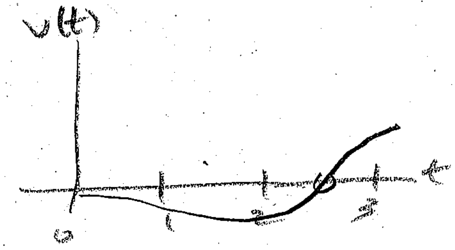
a) $a(t) = -(t+1)\cos\left(\frac{1}{2}t^2\right)t + \sin\left(\frac{1}{2}t^2\right)(-1)$

$\boxed{a(2) = -(2+1)\cos\left(\frac{1}{2}(2)^2\right)(2) + \sin\left(\frac{1}{2}(2)^2\right)(-1)}$

(using a calculator): $a(2) = 1.588 > 0$ and $v(2) = -2.728$

Since $a(2) > 0$, $v(t)$ is increasing, getting less negative, and speed $= |v|$ so $\boxed{\text{speed is decreasing}}$

b) The particle changes direction when the sign of $v(t)$ changes... (must use calculator graph here) ...which occurs at $\boxed{t = 2.507}$



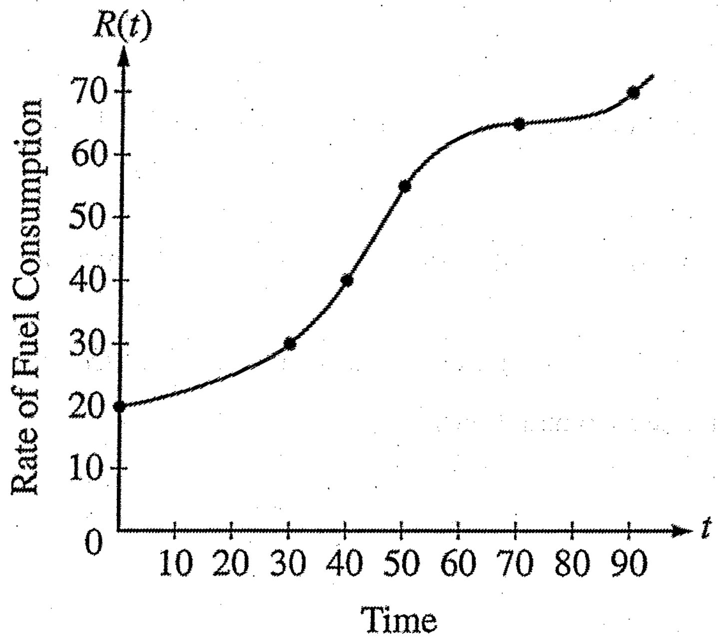
c) total dist $= \int_0^3 |v(t)|\,dt$ (need to separate where $v > 0$ & $v < 0$

total dist $= -\left[\int_0^{2.507} v(t)\,dt\right] + \int_{2.507}^{3} v(t)\,dt$

$= -[-3.2655] + 1.0683$

$= \boxed{4.334}$

#2. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of $R(t)$, for the time interval $0 \le t \le 90$ minutes are shown.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer and indicate units of measure.

b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table.

d) Using your approximation is part (c), how much total fuel was consumed during the entire 90 minute flight?

#2

a) $R'(45) \approx \frac{R(50)-R(40)}{50-40} = \frac{55-40}{10} = \boxed{\frac{15}{10} \ \frac{\text{gallons per minute}}{\text{min}}}$

b) since $R'(t)$ is increasing ~~fastest at~~ $t=45$ min

$R'(t)$ must have a maximum at $t=45$ min,

$\therefore \boxed{R''(45)=0}$

c)

interval	t_i	$R(t_i)$	$\cdot \Delta t$	$=$ area
[0, 30]	0	20	$\cdot$ 30	= 600
[30, 40]	30	30	$\cdot$ 10	= 300
[40, 50]	40	40	$\cdot$ 10	= 400
[50, 70]	50	55	$\cdot$ 20	= 1100
[70, 90]	70	65	$\cdot$ 20	= 1300

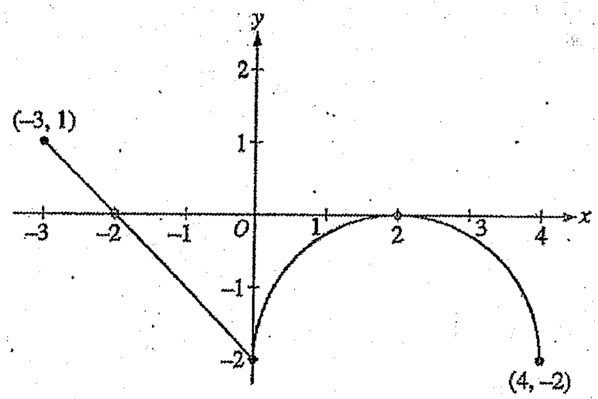
$\int_0^{90} R(t)dt \approx \boxed{3700 \text{ gallons}}$

d) $\int_0^{90} R(t)dt = F(90) - F(0)$

$3700 = F(90) - 0$

$\boxed{F(90) = 3700 \text{ gallons}}$

#3. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of $f'(x)$, the derivative of $f(x)$, consists of one line segment and a semicircle, as shown in the figure.



Graph of f'

a) On what intervals, if any, is f increasing? Justify your answer.

b) Find the x-coordinates of each of the following: critical points, inflections points, and locations where the function may have an absolute maximum or absolute minimum over the interval $-3 \le x \le 4$.

c) Find an equation for the line tangent to the graph of f at x = 0.

d) Find $f(-3)$ and $f(4)$.

a) $f(x)$ increases where $f'(x) > 0$ which occurs for $-3 \le x < 2$

b) critical points when $f'(x) = 0$ or DNE, at $\boxed{x = -2,\ x = 2}$

inflection points when $f''(x) = 0$, which occurs when $f'(x)$ changes from increasing to decreasing or vice versa, at $\boxed{x = 0 \text{ and } x = 2}$

abs max/min at critical points plus interval ends, at $\boxed{x = -2,\ x = 2,\ x = -3,\ x = 4}$

c) point $(0, 3)$ $\quad m = f'(0) = -2$ $\quad \boxed{(y-3) = -2(x-0)}$

d) $\displaystyle\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$

$\frac{1}{2}(1)(1) - \frac{1}{2}(2)(2) = 3 - f(-3)$

$\frac{1}{2} - 2$

$-\frac{3}{2} = 3 - f(-3)$

$\boxed{f(-3) = 3 + \frac{3}{2}}$

$\displaystyle\int_{0}^{4} f'(x)\,dx = f(4) - f(0)$

rectangle − semicircle

$-\left[(4)(2) - \frac{1}{2}\pi(2)^2\right] = f(4) - 3$

$-\left[8 - 2\pi\right] = f(4) - 3$

$\boxed{f(4) = 2\pi - 5}$

#4. Find $\frac{dy}{dx}$ for $y - x^2y^2 = 6$

$$\frac{d}{dx}[y] - [x^2 \frac{d}{dx}[y^2] + y^2 \frac{d}{dx}[x^2]] = \frac{d}{dx}[6]$$

$$\frac{dy}{dx} - [x^2(2y\frac{dy}{dx}) + y^2(2x)] = 0$$

$$\frac{dy}{dx} - 2x^2y\frac{dy}{dx} - 2xy^2 = 0$$

$$\frac{dy}{dx}(1 - 2x^2y) = 2xy^2$$

$$\boxed{\frac{dy}{dx} = \frac{2xy^2}{1-2x^2y}}$$

#5. Find an equation of the tangent line to the curve $y^4 + xy = x^3 - x + 2$ at $(1,1)$.

slope $= \frac{dy}{dx}$

$$\frac{d}{dx}[y^4] + x\frac{d}{dx}[y] + y\frac{d}{dx}[x] = \frac{d}{dx}[x^3] - \frac{d}{dx}[x] + \frac{d}{dx}[2]$$

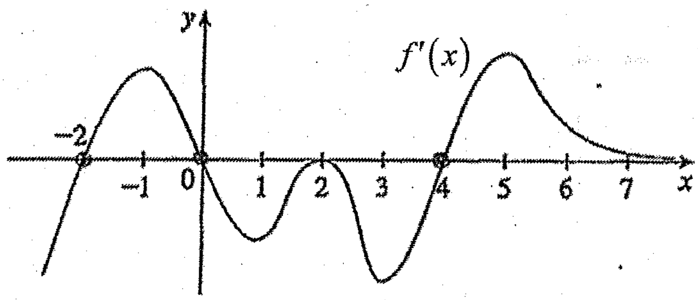
$$4y^3\frac{dy}{dx} + x\frac{dy}{dx} + y(1) = 3x^2 - 1 + 0$$

$$\frac{dy}{dx}[4y^3 + x] = 3x^2 - 1 - y$$

$$m = \frac{dy}{dx} = \left.\frac{3x^2 - 1 - y}{4y^3 + x}\right|_{(1,1)} = \frac{3(1)^2 - 1 - 1}{4(1)^3 + 1} = \frac{1}{5}$$

$$\boxed{(y-1) = \frac{1}{5}(x-1)}$$

#6. The figure shows the graph of $f'(x)$, the derivative of $f(x)$.



Find the following (approximate coordinates to the nearest 0.5):

a) All critical point x-values.
b) x-intervals where $f(x)$ is increasing/decreasing.
c) All inflection point x-values.
d) x-intervals where $f(x)$ is concave up/down.
e) All x-values where $f(x)$ has a local maximum or local minimum.

a) critical points where $f'(x)=0$ or DNE, at $\boxed{x=-2,\ x=0,\ x=2,\ x=4}$

b) $f(x)$ increasing where $f'(x)>0$, which occurs for $\boxed{-2<x<0 \text{ and } 4<x<\infty}$

$f(x)$ decreasing where $f'(x)<0$, which occurs for $\boxed{-\infty<x<-2,\ 0<x<2,\ 2<x<4}$

c) Inflection points where $f''(x)$ changes sign,
where $f'(x)$ changes from increasing / decreasing
or vice versa, which occurs at $\boxed{x=-1,\ x=1,\ x=2,\ x=3,\ x=5}$

d) $f(x)$ is concave up when $f''(x)>0$, when $f'(x)$ is increasing;
which occurs for $\boxed{-\infty<x<-1,\ 1<x<2,\ 3<x<5}$

$f(x)$ is concave down when $f''(x)<0$, when $f'(x)$ is decreasing
which occurs for $\boxed{-1<x<1,\ 2<x<3,\ 5<x<\infty}$

e) Local max or min at critical points (from a): $\boxed{x=-2,\ x=0,\ x=2,\ x=4}$

#7. Given

$$g(x)=\begin{cases} x^2 & x<2 \\ -3 & x=2 \\ 3x & x>2 \end{cases}$$

a) Is $g(x)$ continuous at $x = 2$?

1) $g(2) = -3$

2) $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} x^2 = (2)^2 = 4$

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} 3x = 3(2) = 6$

$\therefore \lim_{x\to 2} g(x)$ DNE

$\boxed{\therefore\ g(x) \text{ is } \underline{\text{not}} \text{ continuous at } x=2}$

b) Is $g(x)$ continuous at $x = 3$?

1) $g(3) = 3(3) = 9$

2) $\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} 3x = 3(3) = 9$

$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} 3x = 3(3) = 9$

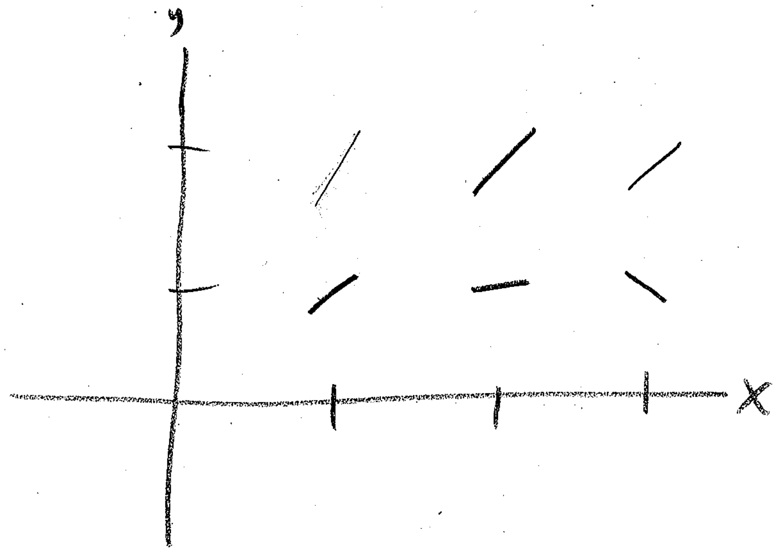
$\therefore \lim_{x\to 3} g(x) = 3$

3) $g(3) = \lim_{x\to 3} g(x)$

$9 = 9$

$\boxed{\therefore\ g(x)\ \underline{\text{is}} \text{ continuous at } x=3}$

#8. Sketch a slope field using at least 6 points in quadrant I for the differential equation $\frac{dy}{dx} = -x + 2y$



(x,y)	$\frac{dy}{dx}$
$(1,1)$	1
$(1,2)$	3
$(2,1)$	0
$(2,2)$	2
$(3,1)$	-1
$(3,2)$	1

Solve the differential equation by separation of variables:

#9. $\frac{dy}{dx} = xy^2$ if $y(1) = 3$.

$$\frac{1}{y^2}dy = x\,dx$$

$$\int y^{-2}dy = \int x\,dx$$

$$\frac{y^{-1}}{-1} = \frac{1}{2}x^2 + c$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + c$$

$$-\frac{1}{(3)} = \frac{1}{2}(1)^2 + c$$

$$-\frac{1}{3} = \frac{1}{2} + c$$

$$c = -\frac{1}{3} - \frac{1}{2} = -\frac{2}{6} - \frac{3}{6} = \frac{-5}{6}$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + \left(\frac{-5}{6}\right)$$

$$\frac{1}{y} = -\frac{1}{2}x^2 + \frac{5}{6}$$

$$\boxed{y = \frac{1}{-\frac{1}{2}x^2 + \frac{5}{6}}}$$

Solve the differential equation by separation of variables:

#10. $\frac{dy}{dx}=(2+x)y^2$ if $y(2)=4$.

$\frac{1}{y^2}\,dy = (2+x)\,dx$

$\int y^{-2}dy = \int (2+x)\,dx$

$\frac{y^{-1}}{-1} = 2x+\frac{1}{2}x^2+C$

$-\frac{1}{y} = 2x+\frac{1}{2}x^2+C$

$-\frac{1}{4} = 2(2)+\frac{1}{2}(2)^2+C$

$-\frac{1}{4} = 4+2+C = 6+C$

$C = -6-\frac{1}{4} = -\frac{24}{4}-\frac{1}{4} = -\frac{25}{4}$

$-\frac{1}{y} = 2x+\frac{1}{2}x^2-\frac{25}{4}$

$\frac{1}{y} = -2x-\frac{1}{2}x^2+\frac{25}{4}$

$$\boxed{y = \frac{1}{-2x-\frac{1}{2}x^2+\frac{25}{4}}}$$

#11. $xyy' = 3+x^2$ if $y(1)=2$.

$y\frac{dy}{dx} = \frac{3+x^2}{x} = \frac{3}{x}+x$

$\int y\,dy = \int \left(\frac{3}{x}+x\right)dx$

$\frac{1}{2}y^2 = 3\ln|x|+\frac{1}{2}x^2+C$

$\frac{1}{2}(2)^2 = 3\ln|1|+\frac{1}{2}(1)^2+C$

$2 = 0+\frac{1}{2}+C, \quad C = 2-\frac{1}{2} = \frac{3}{2}$

so $\frac{1}{2}y^2 = 3\ln|x|+\frac{1}{2}x^2+\frac{3}{2}$

$$\boxed{\begin{array}{l} y^2 = 6\ln|x|+x^2+3 \\ y = \pm\sqrt{6\ln|x|+x^2+3} \end{array}}$$

Multiple-Choice Part: Although these review problems are provided in free-response format with full solution methods posted, on the actual final exam, you'll only need to circle one answer for each problem, no work required. CALCULATOR REQUIRED for the MCQ part of the final exam.

#12. If $f(x)=\frac{e^{5x}}{x^3}$, find $f'(x)$ quotient rule:

$$f'(x)=\frac{x^3(e^{5x}(5))-e^{5x}(3x^2)}{(x^3)^2}$$

(whichever one matches answer)

$$=\frac{5x^3e^{5x}-3x^2e^{5x}}{x^6}$$

$$=\frac{x^2e^{5x}[5x-3]}{x^6}$$

$$=\frac{e^{5x}(5x-3)}{x^4}$$

#13. If $f(x)=\sin(4x)\ln(6x^2+3x)$, find $f'(x)$

product rule: $f'(x)=\sin(4x)\frac{1}{6x^2+3x}(12x+3)+\ln(6x^2+3x)\cos(4x)\cdot 4$

$$=\frac{12x+3}{6x^2+3x}\sin(4x)+4\ln(6x^2+3x)\cos(4x)$$

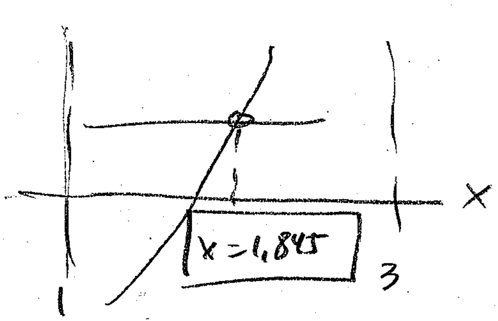
$$=\frac{3(4x+1)}{3x(x+1)}\sin(4x)+4\ln(6x^2+3x)\cos(4x)$$

$$=\frac{4x+1}{x(x+1)}\sin(4x)+4\ln(6x^2+3x)\cos(4x)$$

#14. Let f be the function given by $f(x)=e^x\cos(2x)$. For what value of x in the interval $1\le x\le 3$ is the slope of the tangent line to the graph of f equal to 1.2 ?

$$f'(x)=\underbrace{e^x(-2\sin(2x))+\cos(2x)e^x}_{Y1}=\underbrace{1.2}_{Y2}$$

use calculator graphs



#15. For the function $g(t)=\dfrac{t}{t+2}$

a) Find the average rate of change over the interval $[1,4]$

b) Show that the Mean Value Theorem (or Rolle's Theorem) guarantees a time value within the interval $[1,4]$ where the instantaneous rate of change equals the average rate of change.

c) Find the time value where the instantaneous rate of change equals the average rate of change over the interval $[1,4]$.

a) A.R.o.C $= \dfrac{g(4)-g(1)}{4-1} = \dfrac{\frac{2}{3}-\frac{1}{3}}{3} = \dfrac{\frac{1}{3}}{3} = \frac{1}{3}\cdot\frac{1}{3} = \boxed{\frac{1}{9} = 0.111}$

b) $g(t)$ is continuous over $[1,4]$ and differentiable over $(1,4)$ ∴ the Mean Value Theorem guarantees c, $1<c<4$ such that $g'(c)=\frac{1}{9}$.

c) $g'(t) = \dfrac{(t+2)(1)-t(1)}{(t+2)^2} = \dfrac{t+2-t}{(t+2)^2} = \dfrac{2}{(t+2)^2}$

$g'(c) = \dfrac{2}{(c+2)^2} = \frac{1}{9}$, $(c+2)^2 = 18$

$c+2 = \pm\sqrt{18}$

$c = -2\pm\sqrt{18} = \boxed{-2+\sqrt{18},\ (2,2.43)}$, $\cancel{-2-\sqrt{18}}$ not in $[1,4]$

#16. Evaluate $\displaystyle\lim_{x\to0}\frac{\ln(1-x)+x+\frac{1}{2}x^2}{x^3}$

$\lim_{x\to0} \ln(1-x)+x+\frac{1}{2}x^2 = 0$

$\lim_{x\to0} x^3 = 0$ $\quad (\frac{0}{0})$ use L'Hopital's Rule

$\displaystyle\lim_{x\to0} \frac{\frac{1}{1-x}(-1)+1+x}{3x^2} = \frac{-(1-x)^{-1}+1+x}{3x^2}$ $(\frac{0}{0})$ L'Hop

$\displaystyle\lim_{x\to0} \frac{(1-x)^{-2}(-1)+1}{6x}$ $(\frac{0}{0})$ L'Hop

$\displaystyle\lim_{x\to0} \frac{2(1-x)^{-3}(-1)}{6} = \lim_{x\to0} \frac{-1}{3(1-x)^3} = \boxed{-\frac{1}{3}}$

#17. Evaluate $\displaystyle\lim_{x\to\infty}\frac{\ln(\ln x)}{\ln x}$

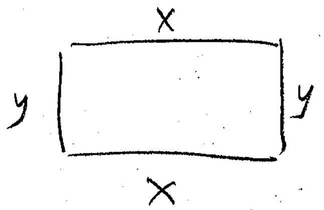
$\lim_{x\to\infty} \ln(\ln x) = \infty$ $\quad (\frac{\infty}{\infty})$ use L'Hopital's Rule

$\lim_{x\to\infty} \ln x = \infty$

$\displaystyle\lim_{x\to\infty} \frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{\ln x} = \boxed{0}$

(could also just graph this in a calculator)

#18. There are 320 yards of fencing available to enclose a rectangular field. How should this fencing be used so that the enclosed area is as large as possible?



objective

max $A = xy$

$A = x(160 - x)$

$A = 160x - x^2$

$A' = 160 - 2x = 0$

$2x = 160$

$x = \frac{160}{2} = 80$ yds

$y = 160 - 80 = 80$ yds

$\boxed{80 \times 80 \text{ yds}}$

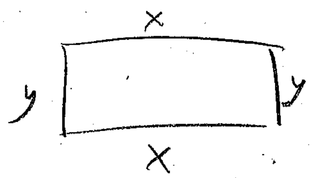
constraint

$2x + 2y = 320$

$x + y = 160$

$y = 160 - x$

#19. A rectangular area of 10,000 m^2 must be enclosed by a fence. What should the dimensions of the rectangle be if we want to enclose this area using the minimum amount of fencing?



objective

min $F = 2x + 2y$

$F = 2x + 2\left(\frac{10000}{x}\right)$

$F = 2x + 20000x^{-1}$

$F' = 2 - 20000x^{-2}$

$F' = 2 - \frac{20000}{x^2} = 0$

$2 = \frac{20000}{x^2}$

$x^2 = \frac{20000}{2} = 10000$

$x = \sqrt{10000} = 100$ m

$y = \frac{10000}{100} = 100$ m

$\boxed{100m \times 100m}$

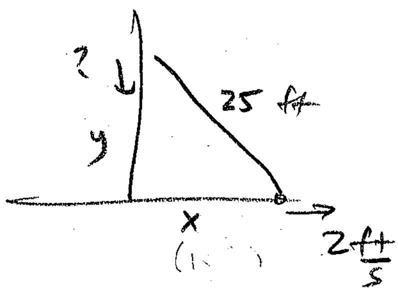
constraint

$A = 10000\ m^2$

$xy = 10000$

$y = \frac{10000}{x}$

#20. A 25-foot long ladder is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second. How fast is the top of the ladder moving down the wall when its base is 15 feet from the wall?



$\frac{dx}{dt} = +2$

$x^2 + y^2 = 25^2$

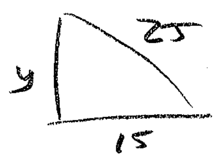
$\frac{d}{dt}[x^2] + \frac{d}{dt}[y^2] = \frac{d}{dt}[25^2]$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

(instantaneous)

$(15)(2) + (20)\frac{dy}{dt} = 0$



$y^2 + 15^2 = 25^2$

$y = \sqrt{25^2 - 15^2} = \sqrt{400} = 20$

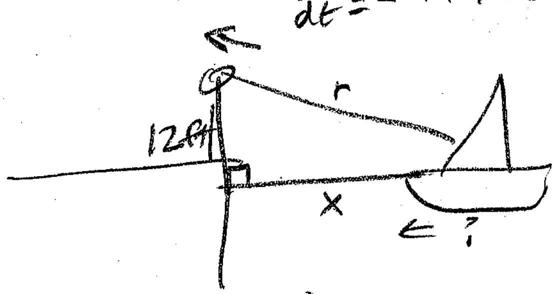
$20\frac{dy}{dt} = -30$

$\frac{dy}{dt} = \frac{-30}{20} = \boxed{-\frac{3}{2} \text{ ft/sec} \quad -1.5 \text{ ft/sec}}$

(could be positive because problem say "moving down")

#21. A boat is pulled into a dock by means of a winch 12 feet above the deck of the boat. Suppose the winch pulls in rope at a rate of 4 feet per second. Determine the speed of the boat moving horizontally when there is 13 feet of rope out.

$\frac{dr}{dt} = -4$ ft/sec



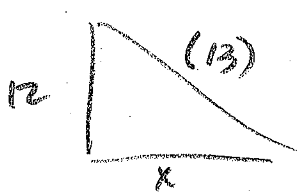
$x^2 + 12^2 = r^2$

$\frac{d}{dt}[x^2] + \frac{d}{dt}[12^2] = \frac{d}{dt}[r^2]$

$2x\frac{dx}{dt} + 0 = 2r\frac{dr}{dt}$

(instantaneous)

$x\frac{dx}{dt} = r\frac{dr}{dt}$



$(5)\frac{dx}{dt} = (13)(-4)$

$\boxed{\frac{dx}{dt} = \frac{13(-4)}{5} \text{ ft/sec}}$

$x^2 + 12^2 = 13^2$

$x = \sqrt{13^2 - 12^2} = \sqrt{25} = 5$ ft

#22. Evaluate $\int \frac{x}{\sqrt{x^7}}\,dx$ $= \int \frac{x}{x^{7/2}}dx = \int x^{-5/2}dx$

(algebra)

$$= \boxed{\frac{x^{-3/2}}{-3/2} + c}$$

$$= \boxed{-\frac{2}{3}x^{-3/2} + c}$$

#23. Evaluate $\int (4x+6)e^{(x^2+3x)}\,dx$

$u = x^2+3x$

$\frac{du}{dx} = 2x+3$

$du = (2x+3)dx$

(u sub)

$\int 2(2x+3)e^{(x^2+3x)}dx$

$2\int e^u du$

$2e^u + c$

$$\boxed{2e^{(x^2+3x)} + c}$$

#24. Evaluate $\int \frac{1}{x(\ln x)^2}\,dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$du = \frac{1}{x}dx$

(u sub)

$\int \frac{1}{u^2}du$

$\int u^{-2}du$

$\frac{u^{-1}}{-1} + c$

$$\boxed{\frac{-1}{\ln x} + c}$$

#25. Evaluate $\int x^2 e^x \, dx$

(by parts)

$u = x^2 \quad dv = e^x dx$

$\frac{du}{dx} = 2x \quad \int dv = \int e^x dx$

$du = 2x\,dx \quad v = e^x$

$uv - \int v\,du$

$x^2e^x - \int e^x 2x\,dx$

$x^2e^x - 2\int xe^x dx$

(by parts again)

$u = x \quad dv = e^x dx$

$\frac{du}{dx} = 1 \quad \int dv = \int e^x dx$

$du = dx \quad v = e^x$

$x^2e^x - 2\left[uv - \int v\,du\right]$

$x^2e^x - 2\left[xe^x - \int e^x dx\right] = x^2e^x - 2\left[xe^x - e^x\right] + C = \boxed{x^2e^x - 2xe^x + 2e^x + C}$

#26. Evaluate $\int x\sin(2x)\,dx$

(by parts)

$u = x \quad dv = \sin(2x)dx$

$\frac{du}{dx} = 1 \quad \int dv = \int \sin(2x)dx$

$du = dx \quad v = -\frac{1}{2}\cos(2x)$

$uv - \int v\,du$

$-\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx$

$\boxed{-\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C}$

#27. Evaluate $\int \frac{1}{(x-4)(x+5)}\,dx = \int \left(\frac{1}{9}\frac{1}{x-4} - \frac{1}{9}\frac{1}{x+5}\right)dx = \boxed{\frac{1}{9}\ln|x-4| - \frac{1}{9}\ln|x+5| + C}$

(partial fractions)

$\frac{1}{(x-4)(x+5)} = \frac{A}{x-4} + \frac{B}{x+5}$

$A(x+5) + B(x-4) = 1$

$Ax + 5A + Bx - 4B = 1$

$(A+B)x + (5A-4B) = (0)x + (1)$

$A + B = 0$

$5A - 4B = 1$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 5 & -4 & 1 \end{array}\right]$

rref

$\left[\begin{array}{cc|c} 1 & 0 & \frac{1}{9} \\ 0 & 1 & -1/9 \end{array}\right]\begin{array}{l} = A \\ = B \end{array}$

#28. Evaluate $\int \frac{1}{x^2-5x-14}\,dx = \int \frac{1}{(x-7)(x+2)}\,dx$ (partial fractions)

$$\frac{1}{(x-7)(x+2)} = \frac{A}{x-7} + \frac{B}{x+2}$$

$$= \int \left(\frac{1}{9}\frac{1}{x-7} - \frac{1}{9}\frac{1}{x+2}\right)dx$$

$$A(x+2) + B(x-7) = 1$$

$$(A+B)x + (2A-7B) = (0)x + (1)$$

$$= \boxed{\frac{1}{9}\ln|x-7| - \frac{1}{9}|x+2| + C}$$

$A+B=0$

$2A-7B=1$

$$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 2 & -7 & 1 \end{array}\right]$$

mef

$$\left[\begin{array}{cc|c} 1 & 0 & 1/9 \\ 0 & 1 & -1/9 \end{array}\right]$$

#29. Evaluate $\int_0^5 \frac{1}{(x-1)^{1/5}}\,dx = \lim_{b\to 1^-} \int_0^b (x-1)^{-1/5}dx + \lim_{c\to 1^+} \int_c^5 (x-1)^{-1/5}dx$

vert Asymptote at $x=1$

$$= \lim_{b\to 1^-} \left[\frac{5}{4}(x-1)^{4/5}\right]_0^b + \lim_{c\to 1^+} \left[\frac{5}{4}(x-1)^{4/5}\right]_c^5$$

$$= \lim_{b\to 1^-} \frac{5}{4}(b-1)^{4/5} - \frac{5}{4}(0-1)^{4/5} + \frac{5}{4}(5-1)^{4/5} - \lim_{c\to 1^+} \frac{5}{4}(c-1)^{4/5}$$

$$= 0 - \frac{5}{4}(-1)^{4/5} + \frac{5}{4}(4)^{4/5} - 0$$

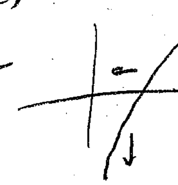
$$= \boxed{2.539}$$

#30. Evaluate $\int_1^3 \frac{2}{x-2}\,dx = \lim_{b\to 2^-} 2\int_1^b \frac{1}{x-2}dx + \lim_{c\to 2^+} 2\int_c^3 \frac{1}{x-2}dx$

vert asymptote at $x=2$

$$\lim_{b\to 2^-} [2\ln|x-2|]_1^b + \lim_{c\to 2^+} [2\ln|x-2|]_c^3$$

$$\lim_{b\to 2^-} 2\ln|b-2| - 2\ln|1-2| + 2\ln|3-2| - \underbrace{\lim_{c\to 2^+} 2\ln|c-2|}_{-\infty}$$



$\boxed{\text{diverges}}$

#31. Evaluate $\int_4^{\infty} \frac{x}{x^{7/2}}\, dx = \lim_{b\to\infty} \int_4^b x^{-5/2}\, dx$

$= \lim_{b\to\infty} \left[-\frac{2}{3} x^{-3/2}\right]_4^b$

$= \lim_{b\to\infty} -\frac{2}{3} \frac{1}{b^{3/2}} - \left[-\frac{2}{3} \frac{1}{(4)^{3/2}}\right]$

$= \quad 0 + \frac{2}{3} \frac{1}{8} = \boxed{\frac{1}{12} = 0.083}$

#32. Find the <u>derivative</u> of the function $F(x) = \int_2^{e^{3x}} \cos(t^2)\, dt$ when $x = 0.2$

$F'(x) = \frac{d}{dx}\left[\int_2^{e^{3x}} \cos(t^2)\, dt\right] = \cos\left((e^{3x})^2\right) e^{(3x)}(3)$

$\boxed{F'(0.2) = \cos\left((e^{3(0.2)})^2\right) e^{(3(0.2))}(3) = -5.379}$

#33. Find the average value of the function $f(x) = -x^4 + 2x^2 + 4$ on the closed interval $[-2, 1]$.

$AV = \frac{1}{b-a} \int_a^b f(x)\, dx$

(math 9)

$= \frac{1}{1-(-2)} \int_{-2}^{1} (-x^4 + 2x^2 + 4)\, dx = \boxed{3.8}$

#34. A particle moves along a line such that its position in meters at time t in seconds is given by $s(t)=2t^3-5t^2+2t$

a) Find the velocity at time t.
b) What is the velocity after 1 second?
c) Find the acceleration at time t.
d) Find the acceleration of the particle at time $t = 4$ seconds.
e) When is the acceleration of the particle zero?
f) When is the speed of the particle 1 $\frac{m}{s}$?
g) At what times in the interval $0 \le t \le 8$ is the particle changing direction?
h) When is the particle at rest?
i) Is the particle moving the right or the left at time $t = 0.4$ seconds? … at time $t = 1.5$ seconds?
j) Is the particle speeding up or slowing down at time $t = 0.4$ seconds? … at time $t = 1.5$ seconds?

a) $\boxed{v(t)=s'(t)=6t^2-10t+2}$

b) $v(1)=6(1)^2-10(1)+2=\boxed{-2\ m/s}$

c) $\boxed{a(t)=v'(t)=12t-10}$

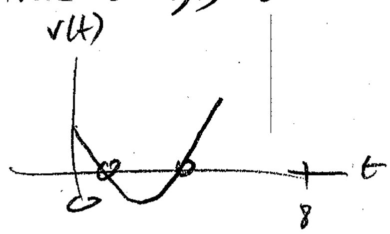
d) $a(4)=12(4)-10=\boxed{38\ m/s^2}$

e) $12t-10=0$
$12t=10$, $\boxed{t=\frac{10}{12}\text{ sec}=0{,}833\text{ sec}}$

f) Speed $=|v|$ $\quad v=\underbrace{6t^2-10t+2}_{Y_1}=\underbrace{1}_{Y_2}$ or $v=\underbrace{6t^2-10t+2}_{Y_1}=\underbrace{-1}_{Y_2}$ (by calc, graph)

$\boxed{\text{at } t=0{,}107\text{ sec and } t=1{,}560\text{ sec}}$ and $\boxed{t=0{,}392\text{ sec and } t=1{,}274\text{ sec}}$

g) particle changes direction when sign of $v(t)$ changes



$\boxed{\text{at } t=0{,}232\text{ sec and } t=1{,}434\text{ sec}}$

(#34 continued)

h) particle is at rest when $v(t)=0$ $\boxed{\begin{array}{l}\text{at } t=0.232\text{ sec}\\ \text{and } t=1.434\text{ sec}\end{array}}$ (by calc, graph)

(i) $v(0.4) = -1.04 < 0$ so particle is $\boxed{\text{moving left at } t=0.4\text{ sec}}$

$v(1.5) = 0.5 > 0$ so particle is $\boxed{\text{moving right at } t=1.5\text{ sec}}$

(j) $v(0.4) = -1.04$ since $v(0.4)<0$ and $a(0.4)<0$ (same sign)

$a(0.4) = -5.2$ the particle is $\boxed{\text{speeding up at } t=0.4\text{ sec}}$

$v(1.5) = 0.5$ since $v(1.5)>0$ and $a(1.5)>0$ (same sign)

$a(1.5) = 8$ the particle is $\boxed{\text{speeding up at } t=1.5\text{ sec}}$

#35. A particle moves along the x-axis such that its acceleration in m/s^2 at time t in seconds is given by $a(t)=t^3-3t$. If $v(2)=5$ and $x(1)=4$, find the velocity and position functions for the particle at any time, t.

$v(t)=\int a(t)dt=\int(t^3-3t)dt$

$v(t)=\frac{1}{4}t^4-\frac{3}{2}t^2+C$, now $v(2)=5$:

$5=\frac{1}{4}(2)^4-\frac{3}{2}(2)^2+C$

$5=-2+C \Rightarrow C=7$

$$\boxed{v(t)=\frac{1}{4}t^4-\frac{3}{2}t^2+7 \quad (m/s)}$$

$x(t)=\int v(t)dt=\int(\frac{1}{4}t^4-\frac{3}{2}t^2+7)dt$

$x(t)=\frac{1}{20}t^5-\frac{1}{2}t^3+7t+D$, now $x(1)=4$

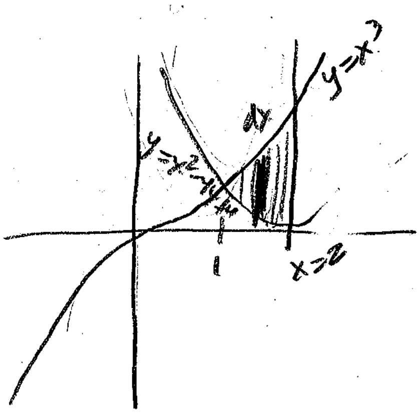
$4=\frac{1}{20}(1)^5-\frac{1}{2}(1)^3+7(1)+D$

$4=\frac{131}{20}+D \Rightarrow D=4-\frac{131}{20}=-\frac{51}{20}$

$$\boxed{x(t)=\frac{1}{20}t^5-\frac{1}{2}t^2+7t-\frac{51}{20} \quad (m)}$$

For #36-37, find the area bounded by the given curves (use math9 to evaluate the integrals)

#36. $y = x^3$, $y = x^2 - 4x + 4$, $x = 2$



$$A = \int_a^b (y_{top} - y_{bottom})\,dx$$

$$A = \int_1^2 [x^3 - (x^2 - 4x + 4)]\,dx = \boxed{3.417}$$

line: $2y = x + 7$

#37. $x - 2y + 7 = 0$, $y^2 - 6y - x = 0$

intersections:

$$\begin{cases} x - 2y + 7 = 0 \\ y^2 - 6y - x = 0 \end{cases}$$

$x = 2y - 7$ into

$y^2 - 6y - (2y - 7) = 0$

$y^2 - 8y + 7 = 0$

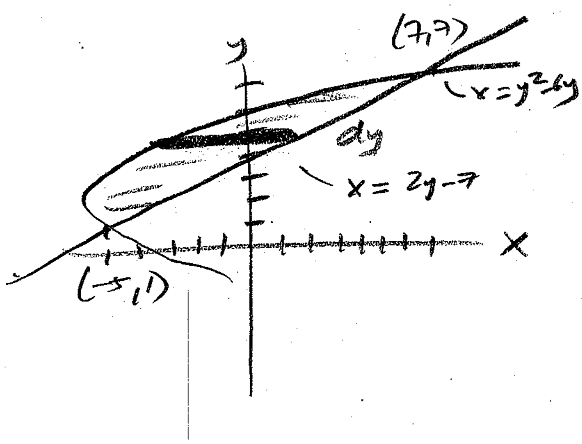
$(y-1)(y-7) = 0$

$y = 1 \quad y = 7$

$x = 2(1) - 7 \quad x = 2(7) - 7$

$x = -5 \quad x = 7$

$(-5, 1) \quad (7, 7)$



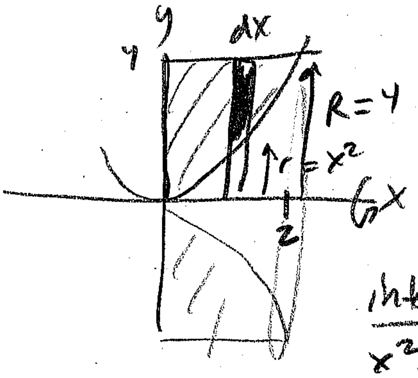
$$A = \int_a^b [x_{top} - x_{bottom}]\,dy$$

$$= \int_1^7 [(2y-7) - (y^2 - 6y)]\,dy$$

$$= \boxed{36}$$

For #38-40, use the disk method to find the volume generated by rotating the region bounded by the given curves about the specified axis (use math9 to evaluate the integrals).

#38) $y = x^2$, $y = 4$, $x = 0$; *about the x – axis*



$$V = \int \pi R^2 dx - \int \pi r^2 dx$$

$$= \int_0^2 \pi (4)^2 dx - \int_0^2 \pi (x^2)^2 dx$$

$$= \pi \int_0^2 [4^2 - (x^2)^2] dx$$

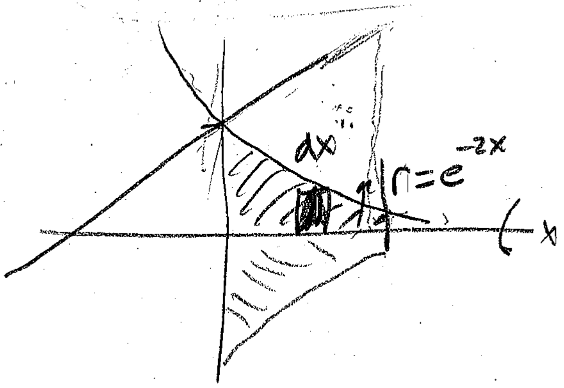
$$= \boxed{80.425}$$

Intersection

$x^2 = 4$

$x = 2$

#39) ~~$y = e^{-2x}$, $y = 1 + x$, $x = 1$, *about the x – axis*~~ $y = e^{-2x}$, $y = 0$, $x = 0$, $x = 1$ about x-axis

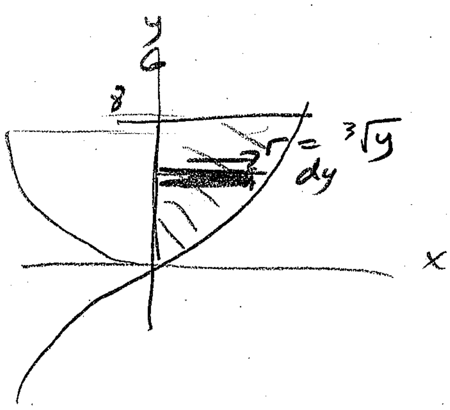


$$A = \int \pi r^2 dx$$

$$= \int_0^1 \pi (e^{-2x})^2 dx$$

$$= \boxed{0.771}$$

#40) $y = x^3$, $y = 8$, $x = 0$, *about the y – axis*



$$A = \int \pi r^2 dy$$

$$= \int_0^8 \pi (\sqrt[3]{y})^2 dy$$

$$= \int_0^8 \pi y^{2/3} dy$$

$$= \boxed{60.319}$$