AP Calculus BC – Study Guide: Unit 1 – Limits and Continuity

Limits and Continuity…

What must be true for $\lim_{x \to c} f(x)$ to exist?

$$\lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$$
$$\textit{where } L \textit{ is a finite number}$$

What must be true for $f(x)$ to be continuous at *c*?

$$1)\ f(c) \textit{ must exist}$$
$$2)\ \lim_{x \to c^-} f(x) = L = \lim_{x \to c^+} f(x)$$
$$\textit{limit must exist}$$
$$3)\ f(c) = L$$

Evaluation tactics…(evaluate these limits):

$\lim_{x \to 2} \frac{x-3}{x^2-7}$

Plug in:

$$\lim_{x \to 2} \frac{x-3}{x^2-7} = \frac{(2)-3}{(2)^2-7} = \frac{-1}{-3} = \frac{1}{3}$$

$\lim_{x \to 5} \frac{x^2-25}{x-5}$

Factor and cancel:

$$\lim_{x \to 5} \frac{x^2-25}{x-5} \left(\frac{0}{0}\right) = \lim_{x \to 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x \to 5} (x+5) = 10$$

$\lim_{x \to 9} \frac{x^2-81}{\sqrt{x}-3}$

Rationalize:

$$\lim_{x \to 9} \frac{x^2-81}{\sqrt{x}-3} = \lim_{x \to 9} \frac{(x^2-81)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}$$
$$= \lim_{x \to 9} \frac{(x-9)(x+9)(\sqrt{x}+3)}{x-9} = \lim_{x \to 9} (x+9)(\sqrt{x}+3) = (18)(6)$$

What is L'Hopital's Rule?

$$\textit{If } \lim_{x \to c} \frac{f(x)}{g(x)} \textit{ is indeterminant form } \frac{0}{0} \textit{ or } \frac{\pm\infty}{\pm\infty}$$
$$\textit{then } \lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

Evaluate using L'Hopital's rule:

$$\lim_{x\to\infty}\frac{2x^2-x}{x^2+x}$$

$$\lim_{x\to\infty}\frac{2x^2-x}{x^2+x}\left(\frac{\infty}{\infty}\right)$$
$$=\lim_{x\to\infty}\frac{4x-1}{2x+1}=\lim_{x\to\infty}\frac{4}{2}=2$$

$$\lim_{x\to\infty}e^{-x}\sqrt{x}$$

$$\lim_{x\to\infty}e^{-x}\sqrt{x}\ (0\bullet\infty)$$
$$=\lim_{x\to\infty}\frac{\sqrt{x}}{e^x}\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{\frac{1}{2}x^{-\frac{1}{2}}}{e^x}=\lim_{x\to\infty}\frac{1}{2\sqrt{x}e^x}=\frac{1}{\infty}=0$$

$$\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\cot x}$$

function raised to a function? Ln of both sides…

$$y=\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\cot x}$$
$$\ln(y)=\ln\left(\lim_{x\to 0}\left(1+\frac{x}{2}\right)^{\cot x}\right)=\lim_{x\to 0}\left[\ln\left(\left(1+\frac{x}{2}\right)^{\cot x}\right)\right]$$
$$\ln(y)=\lim_{x\to 0}\left[\cot(x)\ln\left(\left(1+\frac{x}{2}\right)\right)\right]\left(\frac{\cos 0}{\sin 0}\ln 1=\infty\bullet 0\right)$$
$$\ln(y)=\lim_{x\to 0}\left[\frac{\ln\left(\left(1+\frac{x}{2}\right)\right)}{\tan(x)}\right]\left(\frac{0}{0}\right)\ l'Hopital's\ rule...$$
$$\ln(y)=\lim_{x\to 0}\left[\frac{\frac{1}{1+\frac{x}{2}}\left(\frac{1}{2}\right)}{\sec^2(x)}\right]=\frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{(\cos 0)^2}\right)}=\frac{1}{2}$$
$$y=e^{\frac{1}{2}}=\sqrt{e}$$

Special memorized limits:

$$\lim_{x\to 0}\frac{\sin x}{x}=$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

Horizontal asymptotes occur when…

Vertical asymptotes occur when…

Horizontal asymptotes occur when…

$$\lim_{x\to\pm\infty}f(x)=any\ constant$$

Vertical asymptotes occur when…

$$\lim_{x\to c}f(x)=\pm\infty$$

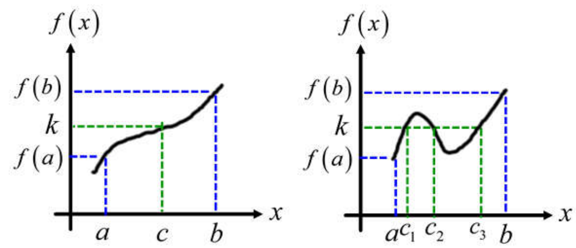
(whenever the function's y value is approaching infinity as x approaches a number – usually at uncancelled zeros in the denominator of rational functions)

Important Theorems…

What is the Intermediate Value Theorem?

<u>Intermediate Value Theorem</u>

If f *is continuous on* $[a,b]$, $f(a) \neq f(b)$, *and* k *is any numberbetween* $f(a)$ *and* $f(b)$, *then there is at least one number* c *in* $[a,b]$ *such that* $f(c) = k$.



Note: This theorem doesn't provide a method for finding the value(s) *c*, and doesn't indicate the number of *c* values which map to *k*, it only guarantees the existence of at least one number *c* such that f(c) = k.

What is the Squeeze Theorem?

<u>Squeeze Theorem</u>

If $h(x) \leq f(x) \leq g(x)$ *for all* x *in an open interval containing* c, *except possibly at* c *itself, and if* $\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$ *then* $\lim_{x \to c} f(x)$ *exists and is equal to* L.