AP Calculus BC – Study Guide: Unit 2 – Derivative Evaluation

Derivatives…

Average rate of change of $f(x) =$

(from x = a to x = b)

Average rate of change of $f(x) = \dfrac{f(b) - f(a)}{b - a}$

Instantaneous rate of change of $f(x)$ at x is…

Instantaneous rate of change of $f(x)$ at x $f'(x)$

Limit definition of derivative, $f'(x) =$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$-or-$$

$$f'(a) = \lim_{x \to a} \frac{f(x+a) - f(a)}{x - a}$$

Notation forms for first derivatives:

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$$y', \quad f'(x), \quad \frac{dy}{dx}, \quad \frac{d}{dx}[y], \quad D_x(y)$$

Notation forms for higher-order derivatives:

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$$y'', \quad f''(x), \quad \frac{d^2 y}{dx^2}$$

$$y''', \quad f'''(x), \quad \frac{d^3 y}{dx^3}$$

$$y^{(4)}, \quad f^{(4)}(x), \quad \frac{d^{(4)} y}{dx^{(4)}}$$

$$\ldots \quad \ldots \quad \ldots$$

$$y^{(n)}, \quad f^{(n)}(x), \quad \frac{d^{(n)} y}{dx^{(4n)}}$$

Derivative shortcuts…

$\frac{d}{dx}[c]=$ $\frac{d}{dx}[c]=0$

$\frac{d}{dx}\left[x^n\right]=$ $\frac{d}{dx}\left[x^n\right]=nx^{n-1}$

$\frac{d}{dx}\left[e^x\right]=$ $\frac{d}{dx}\left[e^x\right]=e^x$

$\frac{d}{dx}\left[a^x\right]=$ $\frac{d}{dx}\left[a^x\right]=a^x\ln(a)$

$\frac{d}{dx}\left[\ln(x)\right]=$ $\frac{d}{dx}\left[\ln(x)\right]=\frac{1}{x}$

$\frac{d}{dx}\left[\log_b(x)\right]=$ $\frac{d}{dx}\left[\log_b(x)\right]=\frac{1}{x\ln(b)}$

$\frac{d}{dx}\left[\sin(x)\right]=$ $\frac{d}{dx}\left[\sin(x)\right]=\cos(x)$

$\frac{d}{dx}\left[\cos(x)\right]=$ $\frac{d}{dx}\left[\cos(x)\right]=-\sin(x)$

$\frac{d}{dx}\left[\tan(x)\right]=$ $\frac{d}{dx}\left[\tan(x)\right]=\sec^2(x)$

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$\frac{d}{dx}\left[\sec(x)\right]=$ $\frac{d}{dx}\left[\sec(x)\right]=\sec(x)\tan(x)$

$\frac{d}{dx}\left[\csc(x)\right]=$ $\frac{d}{dx}\left[\csc(x)\right]=-\csc(x)\cot(x)$

$\frac{d}{dx}\left[\cot(x)\right]=$ $\frac{d}{dx}\left[\cot(x)\right]=-\csc^2(x)$

$\frac{d}{dx}\left[\sin^{-1}(x)\right]=$ $\frac{d}{dx}\left[\sin^{-1}(x)\right]=\frac{1}{\sqrt{1-x^2}}$ $\left(\frac{d}{dx}\left[\cos^{-1}(x)\right]=\frac{-1}{\sqrt{1-x^2}}\right)$

$\frac{d}{dx}\left[\tan^{-1}(x)\right]=$ $\frac{d}{dx}\left[\tan^{-1}(x)\right]=\frac{1}{1+x^2}$ $\left(\frac{d}{dx}\left[\cot^{-1}(x)\right]=\frac{-1}{1+x^2}\right)$

$\frac{d}{dx}\left[\sec^{-1}(x)\right]=$ $\frac{d}{dx}\left[\sec^{-1}(x)\right]=\frac{1}{|x|\sqrt{x^2-1}}$ $\left(\frac{d}{dx}\left[\csc^{-1}(x)\right]=\frac{-1}{|x|\sqrt{x^2-1}}\right)$

Derivative properties/procedures…

$\frac{d}{dx}[cx]=$

$\frac{d}{dx}[cx]=c\frac{d}{dx}[x]$ (constants can be moved out)

$\frac{d}{dx}[f(x)\pm g(x)]=$

$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$

(derivative of each term separately)

$\frac{d}{dx}[f(x)g(x)]=$ (product rule)

$\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$

(1st times deriv. of 2nd plus 2nd times deriv. of 1st)

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=$ (quotient rule)

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

(low-dhigh minus high-dlow over low squared)

$\frac{d}{dx}[f(g(x))]=$ (chain rule)

$\frac{d}{dx}[f(g(x))]=f'(g(x))\bullet g'(x)$

(deriv. of outside (with same inside) times deriv. of inside)

1) Implicit differentiation:

ex: Find $\frac{dy}{dx}$ $for\ xy^3+3x^2=4-y^5$

1) Implicit differentiation:

$$x\frac{d}{dx}[y^3]+y^3\frac{d}{dx}[x]+\frac{d}{dx}[3x^2]=\frac{d}{dx}[4]-\frac{d}{dx}[y^5]$$

$$x\left(3y^2\frac{dy}{dx}\right)+y^3(1)+6x=0-5y^4\frac{dy}{dx}$$

$$\frac{dy}{dx}(3xy^2+5y^4)=-6x-y^3$$

$$\frac{dy}{dx}=\frac{-6x-y^3}{3xy^2+5y^4}$$

2) Logarithmic differentiation:

ex: Find $\frac{dy}{dx}$ $for\ \ y=x^{(5x^3+2x)}$

2) Logarithmic differentiation:

$$\ln(y)=\ln\left(x^{5x^3+2x}\right)$$

$$\ln(y)=(5x^3+2x)\ln(x)$$

$$\frac{d}{dx}[\ln(y)]=\frac{d}{dx}[(5x^3+2x)\ln(x)]$$

$$\frac{d}{dx}[\ln(y)]=(5x^3+2x)\frac{d}{dx}[\ln(x)]+\ln(x)\frac{d}{dx}[(5x^3+2x)]$$

$$\frac{1}{y}\frac{dy}{dx}=(5x^3+2x)\frac{1}{x}+\ln(x)(15x^2+2)$$

$$\frac{dy}{dx}=\left[(5x^3+2x)\frac{1}{x}+\ln(x)(15x^2+2)\right]y$$

$$\frac{dy}{dx}=\left[(5x^3+2x)\frac{1}{x}+\ln(x)(15x^2+2)\right]x^{(5x^3+2x)}$$